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On some new generalized difference vector-valued sequence spaces defined by a sequence of modulus functions

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Abstract: In the present paper we introduce some difference vector-valued sequence spaces defined by a sequence of modulus functions and a multiplier sequence $u = (u_k)$ of non-zero complex numbers. We also make an efforts to study some topological properties and inclusion relation between these spaces. It is also shown that if a sequence is strongly $\Delta_n^m u_q$-Cesaro summable with respect to the modulus function f then it is $\Delta_n^m u_q$-statistically convergent.

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1. Introduction and preliminaries

The studies on vector-valued sequence spaces was exploited by Kamthan [11], Ratha and Srivastava [18], Leonard [14], Gupta [9], Tripathy and Sen [24] and many others. The scope for the studies on sequence spaces was extended on introducing the notion of associated multiplier sequences. Goes and Goes [8] defined the differentiated sequence space dE and integrated sequence space $\int E$ for a given sequence space E, with the help of multiplier sequences (k^{-1}) and (k) respectively. Kamthan used the multiplier sequence $(k!)$ see [11]. Studies on multiplier sequence spaces were carried out by Colak [1], Colak et al. [4], Srivastava and Srivastava [23], Tripathy and Mahanta [25] and many others.

Let w be the set of all sequences of real or complex numbers and let l_∞, c and c_0 be the Banach spaces of Bounded, convergent and null sequences $x = (x_k)$ respectively with the usual norm $\|x\| = \sup |x_k|$, where $k \in \mathbb{N}$, is the set of positive integers.

Throughout the paper, for all $k \in \mathbb{N}, E_k$ are seminormed spaces seminormed by q_k and X is a seminormed space seminormed by q. By $w(E_k), c(E_k), l_\infty(E_k)$ and $l_p(E_k)$ we denote the spaces of all convergent, bounded and p-absolutely summable E_k-valued sequences. In case $E_k = \mathbb{C}$ (the field of complex numbers) for all $k \in \mathbb{N}$, one has the scalar valued sequence spaces. The zero elements of E_k are denoted by θ_k. The zero sequence is denoted by $\bar{\theta} = (\theta_k)$.

A function $f : [0, \infty) \to [0, \infty)$ is said to be a modulus function if it satisfies the following:

(1) $f(x) = 0$ if and only if $x = 0$;
(2) $f(x + y) \leq f(x) + f(y)$ for all $x \geq 0$, $y \geq 0$;
(3) f is increasing;
(4) f is continuous from right at 0.

It follows that f must be continuous everywhere on $[0, \infty)$. The modulus function may be bounded or unbounded. For example, if we take $f(x) = \frac{x}{x+1}$, then $f(x)$ is bounded. If $f(x) = x^p$, $0 < p < 1$, then $f(x)$ is unbounded. Subsequentially, modulus function has been discussed in ([3], [16], [19], [21],) and many others.
Let X be a linear metric space. A function $p : X \to \mathbb{R}$ is called paranorm, if

(1) $p(x) \geq 0$, for all $x \in X$,
(2) $p(-x) = p(x)$, for all $x \in X$,
(3) $p(x + y) \leq p(x) + p(y)$, for all $x, y \in X$,
(4) if (λ_n) is a sequence of scalars with $\lambda_n \to \lambda$ as $n \to \infty$ and (x_n) is a sequence of vectors with $p(x_n - x) \to 0$ as $n \to \infty$, then $p(\lambda_n x_n - \lambda x) \to 0$ as $n \to \infty$.

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called total paranorm and the pair (X, p) is called a total paranormed space. It is well known that the metric of any linear metric space is given by some total paranorm (see [26], Theorem 10.4.2, P-183).
The notion of difference sequence spaces was introduced by Kizmaz [12], who studied the difference sequence spaces $l_\infty(\Delta)$, $c(\Delta)$ and $c_o(\Delta)$. The notion was further generalized by Et and Colak [4] by introducing the spaces $l_\infty(\Delta^m), c(\Delta^m)$ and $c_o(\Delta^m)$.
Let m, n be non-negative integers, then for $Z = l_\infty,\ c,\ c_0$. We have sequence spaces

$$Z(\Delta_n^m) = \{x = (x_k) \in w : (\Delta_n^m x_k) \in Z\},$$

where $\Delta_n^m x = (\Delta_n^m x_k) = (\Delta_n^{m-1} x_k - \Delta_n^{m-1} x_{k+1})$ and $\Delta_n^0 x_k = x_k$ for all $k \in \mathbb{N}$, which is equivalent to the following binomial representation

$$\Delta_n^m x_k = \sum_{v=0}^{m} (-1)^v \begin{pmatrix} m \\ v \end{pmatrix} x_{k+nv}. \tag{1.1}$$

Taking $n = 1$, we get the spaces which were studied by Et and Colak [4]. Taking $m = n = 1$, we get the spaces which were introduced and studied by Kizmaz [12].
The following inequality will be used throughout the paper. Let $p = (p_k)$ be a sequence of positive real numbers with $0 \leq p_k \leq \sup p_k = G$, $K = \max(1, 2^{G-1})$ then

$$|a_k + b_k|^{p_k} \leq K\{|a_k|^{p_k} + |b_k|^{p_k}\} \tag{1.2}$$

for all k and $a_k, b_k \in \mathbb{C}$. Also $|a|^{p_k} \leq \max(1, |a|^G)$ for all $a \in \mathbb{C}$.

Let (E_k, q_k) be a sequence of seminormed spaces such that $E_{k+1} \subset E_k$ for each $k \in \mathbb{N}, p = (p_k)$ a sequence of strictly positive real numbers, $Q = (q_k)$ a sequence of seminorms, $F = (f_k)$ a sequence of modulus functions, and $u = (u_k)$ any fixed sequence of nonzero complex numbers. In the present paper we define the following classes of sequences:

$$w_0(\Delta_n^m, F, Q, p, u) = \Big\{x = (x_k) : x_k \in E_k : \frac{1}{n} \sum_{k=1}^{n} [f_k(q_k(u_k \Delta_n^m x_k))]^{p_k} \to 0, \text{ as } n \to \infty\Big\},$$

$$w_1(\Delta_n^m, F, Q, p, u) = \Big\{x = (x_k) : x_k \in E_k : \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k - l))]^{p_k} \to 0, \text{ as } n \to \infty, \\ l \in E_k\Big\}$$

and

$$w_\infty(\Delta_n^m, F, Q, p, u) = \Big\{x = (x_k) : x_k \in E_k : \sup_n \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k))]^{p_k} < \infty\Big\} \quad (1.3)$$

Throughout the paper z will denote any one of the notation $0, 1$ or ∞. If $f_k = f$ and $q_k = q$ for all $k \in \mathbb{N}$, we will write $w_z(\Delta_n^m, f, q, p, u)$ instead of $w_z(\Delta_n^m, F, Q, p, u)$. If $f(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, we will write $w_z(\Delta_n^m, q, u)$ instead of $w_z(\Delta_n^m, f, q, p, u)$.
If $x \in w_1(\Delta_n^m, f, q, p, u)$ we say that x is strongly $\Delta_n^m u_q$-Cesaro summable with respect to the modulus function f and we will write $x_k \to l(w_1(\Delta_n^m, f, q, p, u))$, where l will be called $\Delta_n^m u_q$-limit of x with respect to the modulus function f.
The aim of this paper is to study some topological properties and some inclusion relation between the above defined classes of sequences $w_z(\Delta_n^m, F, Q, p, u)$.

2. Main Results

Theorem 2.1. Let the sequence $p = (p_k)$ be bounded. Then the spaces $w_z(\Delta_n^m, F, Q, p, u)$ are linear spaces.
Proof. We shall prove the result for $z = 0$. let $x = (x_k), y = (y_k) \in w_0(\Delta_n^m, F, Q, p, u)$. and $\alpha, \beta \in \mathbb{C}$. Then there exist integers M_α and N_β such that $|\alpha| \leq M_\alpha$ and $|\beta| \leq N_\beta$. By using inequality (1.2) and the properties of modulus function, we have

$$\begin{aligned}
\frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m(\alpha x_k + \beta y_k)))]^{p_k} &\leq \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(\alpha u_k\Delta_n^m x_k + \beta u_k\Delta_n^m y_k))]^{p_k} \\
&\leq D\frac{1}{n}\sum_{k=1}^{n}[M_\alpha f_k(q_k(u_k\Delta_n^m x_k))]^{p_k} \\
&+ D\frac{1}{n}\sum_{k=1}^{n}[N_\beta f_k(q_k(u_k\Delta_n^m y_k))]^{p_k} \\
&\leq DM_\alpha^H\frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k))]^{p_k} \\
&+ DN_\beta^H\frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m y_k))]^{p_k} \\
&\to 0 \text{ as } n \to \infty.
\end{aligned}$$

This proves that $w_0(\Delta_n^m, F, Q, p, u)$ is a linear space. Similarly we can prove that $w_1(\Delta_n^m, F, Q, p, u)$ and $w_\infty(\Delta_n^m, F, Q, p, u)$ are linear spaces.

Theorem 2.2. Let f be a modulus function and let $p = (p_k)$ be a bounded sequence. Then

$$w_0(\Delta_n^m, F, Q, p, u) \subset w_1(\Delta_n^m, F, Q, p, u) \subset w_\infty(\Delta_n^m, F, Q, p, u) \quad (2.1)$$

and the inclusions are strict.
Proof. The inclusion $w_0(\Delta_n^m, F, Q, p, u) \subset w_1(\Delta_n^m, F, Q, p, u)$ is obvious. Now, let

$x = (x_k) \in w_1(\Delta_n^m, F, Q, p, u)$ then

$$\frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k))]^{p_k} \to 0, \text{ as } n \to \infty.$$

Now by using (1.2) and the properties of modulus function, we have

$$\begin{aligned}
\sup_n \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k))]^{p_k} &= \sup_n \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k - l + l))]^{p_k} \\
&\leq D\sup_n \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k - l))]^{p_k} \\
&+ D\sup_n \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(l))]^{p_k} \\
&\leq D\sup_n \frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k - l))]^{p_k} \\
&+ D\max\{f_k(q_k(l))^h, f_k(q_k(l))^H\} \\
&< \infty.
\end{aligned}$$

Hence $x = (x_k) \in w_\infty(\Delta_n^m, F, Q, p, u)$. This proves that $w_1(\Delta_n^m, F, Q, p, u) \subset w_\infty(\Delta_n^m, F, Q, p, u)$. This completes the proof of the theorem.

Theorem 2.3. Let $p = (p_k)$ be a bounded sequence of positive real numbers.Then the space $w_0(\Delta_n^m, F, Q, p, u)$ is a complete paranormed space with the paranorm defined by

$$g_\Delta(x) = \sup_n \Big(\frac{1}{n}\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m x_k))]^{p_k}\Big)^{\frac{1}{M}}, \tag{2.2}$$

where $M = \max(1, \sup p_k)$.

Proof. Let $(x^{(i)})$ be a cauchy sequence in $w_0(\Delta_n^m, F, Q, p, u)$. Then for a given $\epsilon > 0$, there exists n_0 such that $g(x^i - x^j) < \epsilon$, for all $i, j \geq n_0$. Thus we have

$$\Big[\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m(x_k^i - x_k^j)))]^{p_k}\Big]^{\frac{1}{M}} < \epsilon, \text{ for all } i, j \geq n_0. \tag{2.3}$$

$$\Rightarrow \Big(f_k(q_k(u_k\Delta_n^m(x_k^i - x_k^j)))\Big) < \epsilon, \text{ for all } i, j \geq n_0.$$

$$\Rightarrow \Delta_n^m(x_k^i - x_k^j) < \epsilon, \text{ for all } i, j \geq n_0, \text{ for all } k \in \mathbb{N}.$$

Hence $(x_k^i)_{i=1}^\infty$ is a cauchy sequence in E_k, for each $k \in \mathbb{N}$. Since E_k are complete for each $k \in \mathbb{N}$, so $(x_k^i)_{i=1}^\infty$ converges in E_k, for each $k \in \mathbb{N}$. On taking limit as $j \to \infty$ in (2.3), we have

$$\Big[\sum_{k=1}^{n}[f_k(q_k(u_k\Delta_n^m(x_k^i - x_k)))]^{p_k}\Big]^{\frac{1}{M}} < \epsilon, \text{ for all } i \geq n_0.$$

$$\Rightarrow \Delta_n^m(x_k^i - x) \in w_0(\Delta_n^m, F, Q, p, u).$$

Since $w_0(\Delta_n^m, F, Q, p, u)$ is a linear space, so we have $x = x^{(i)} - (x^{(i)} - x) \in w_0(\Delta_n^m, F, Q, p, u)$. Thus $w_0(\Delta_n^m, F, Q, p, u)$ is a complete paranormed space. This completes the proof of the theorem.

Theorem 2.4. Let $F = (f_k)$ and $G = (g_k)$ be any two sequences of modulus functions. For any bounded sequences $p = (p_k)$ and $t = (t_k)$ of strictly positive real numbers and for any two sequences of seminorms $Q = (q_k)$ and $R = (r_k)$, we have
(i) $w_z(\Delta_n^m, f, Q, u) \subset w_z(\Delta_n^m, fog, Q, u)$;
(ii) $w_z(\Delta_n^m, F, Q, p, u) \cap w_z(\Delta_n^m, F, R, p, u) \subset w_z(\Delta_n^m, F, Q + R, p, u)$;
(iii) $w_z(\Delta_n^m, F, Q, p, u) \cap w_z(\Delta_n^m, G, Q, p, u) \subset w_z(\Delta_n^m, F + G, Q, p, u)$;
(iv) If q_k is stronger than r_k for each $k \in \mathbb{N}$, then $w_z(\Delta_n^m, F, Q, p, u) \subset w_z(\Delta_n^m, F, R, p, u)$;
(v) If q_k equivalent to r_k for each $k \in \mathbb{N}$, then $w_z(\Delta_n^m, F, Q, p, u) = w_z(\Delta_n^m, F, R, p, u)$;
(vi) $w_z(\Delta_n^m, F, Q, p, u) \cap w_z(\Delta_n^m, F, R, p, u) \neq \phi$.
Proof. (i) We shall prove (i) for the case $z = 0$. We choose $\delta, 0 < \delta < 1$, such that $f(t) < \epsilon$ for $0 \leq t \leq \delta$ and all $k \in \mathbb{N}$. We write $y_k = g(q_k(u_k \Delta_n^m x_k))$ and consider

$$\sum_{k=1}^{n} [f(y_k)] = \sum_{1} [f(y_k)] + \sum_{2} [f(y_k)], \tag{2.4}$$

where the first summation is over $y_k \leq \delta$ and the second summation is over $y_k > \delta$. Since f is continuous, we have

$$\sum_{1} [f(y_k)] < n\epsilon. \tag{2.5}$$

By the definition of f, we have the following relation for $y_k > \delta$,

$$f(y_k) < 2f(1)\frac{y_k}{\delta}. \tag{2.6}$$

Hence

$$\frac{1}{n}\sum_{2} [f(y_k)] \leq 2\delta^{-1} f(1) \frac{1}{n} \sum_{k=1}^{n} y_k. \tag{2.7}$$

It follows from equations (2.5) and (2.7) that $w_z(\Delta_n^m, f, Q, u) \subset w_z(\Delta_n^m, fog, Q, u)$. Similarly, we can prove the result for other cases.
corollary 2.5. Let f be a modulus function. Then $w_z(\Delta_n^m, Q, u) \subset w_z(\Delta_n^m, f, Q, u)$.
Proof. It is easy to prove in view of theorem 2.4(i).

Theorem 2.6. Let $0 < p_k \leq r_k$ and $\left(\frac{r_k}{p_k}\right)$ be bounded. Then

$$w_z(\Delta_n^m, F, Q, r, u) \subset w_z(\Delta_n^m, F, Q, p, u).$$

Proof. By taking $y_k = [f_k(q_k(u_k \Delta_n^m x_k))]^{r_k}$ for all k and using the same technique of Maddox ([15], thm.5) one can easily prove the theorem.

Theorem 2.7. Let f be a modulus function. If $\lim_{s \to \infty} \dfrac{f(s)}{s} = \alpha > 0$, then

$$w_1(\Delta_n^m, Q, p, u) = w_1(\Delta_n^m, f, Q, p, u).$$

Proof. It is easy to prove so we omit the details.

3. $\Delta_n^m u_q$-Statistical Convergence

The notion of statistical convergence were introduced by Fast [6] and Schoenberg [22], independently. Over the years and under different names, statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. later on, it was further investigated from the sequence point of view and linked with summability theory by Fridy [7], Connor [5], Salat [20], Mursaleen [17], Isik [10], Savas [21], Malkosky and Savas [16], Kolk [13], Maddox [15], Tripathy and Sen [24] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Cech compactification of natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability. The notion depends on the density of subsets of the set $\mathbb{N}$ of natural numbers.
A subset E of $\mathbb{N}$ is said to have the natural density $\delta(E)$ if the following limit exists:

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k) \tag{3.1}$$

where χ_E is the characteristic function of E. It is clear that any finite subset of $\mathbb{N}$ have zero natural density and $\delta(E^c) = 1 - \delta(E)$.
In this section, we introduce $\Delta_n^m u_q$-statistically convergent sequences and give some relation between $\Delta_n^m u_q$-statistically convergent sequences and $w_1(f, q, p, u)$-summable sequences.
Definition A sequence $x = (x_k)$ is said to be $\Delta_n^m u_q$-statistically convergent to l if for every $\epsilon > 0$,

$$\delta\Big(k \in \mathbb{N} : q(u_k \Delta_n^m x_k - l) \geq \epsilon\Big) = 0. \tag{3.2}$$

In this case, we write $x_k \to l(S_u^q(\Delta_n^m))$. The set of all $\Delta_n^m u_q$-statistically convergent sequences is denoted by $S_u^q(\Delta_n^m)$. In case $l = 0$, we write $S_{0u}^q(\Delta_n^m)$ instead of $S_u^q(\Delta_n^m)$.

Theorem 3.1. Let f be a modulus function. Then
(i) If $x_k \to l(w_1(\Delta_n^m, q, u))$, then $x_k \to l(S_u^q(\Delta_n^m))$;
(ii) If $x \in l_\infty(\Delta_n^m u_q)$ and $x_k \to l(S_u^q(\Delta_n^m))$, then $x_k \to l(w_1(\Delta_n^m, q, u))$;
(iii) $S_u^q(\Delta_n^m) \cap l_\infty(\Delta_n^m u_q) = w_1(\Delta_n^m, q, u) \cap l_\infty(\Delta_n^m u_q)$,
where $l_\infty(\Delta_n^m u_q) = \Big\{x \in w(X) : \sup_k q(u_k \Delta_n^m x_k) < \infty\Big\}$.
Proof. The proof is easy so we omit the details.

Theorem 3.2. let $p = (p_k)$ be a bounded sequence and $0 < h = \inf p_k \leq p_k \leq \sup p_k \leq \sup p_k = H < \infty$ and let f be a modulus function. Then

$$w_1(\Delta_n^m, f, q, p, u) \subset S_u^q(\Delta_n^m).$$

Proof. Let $x \in w_1(\Delta_n^m, f, q, p, u)$ and let $\epsilon > 0$ be given. Let $\sum_1$ and $\sum_2$ denote the sums over $k \leq n$ with $q(u_k \Delta_n^m x_k - l) \geq \epsilon$ and $q(u_k \Delta_n^m x_k - l) < \epsilon$, respectively.

Then

$$\begin{aligned}\frac{1}{n}\sum_{k=1}^{n}[f(q(u_k\Delta_n^m x_k-l))]^{p_k} &= \frac{1}{n}\sum_{1}[f(q(u_k\Delta_n^m x_k-l))]^{p_k}\\ &\geq \frac{1}{n}\sum_{1}[f(\epsilon)]^{p_k}\\ &\geq \frac{1}{n}\sum_{1}\min([f(\epsilon)]^h,[f(\epsilon)]^H) \qquad (3.3)\\ &\geq \frac{1}{n}|\{k\leq n: q(u_k\Delta_n^m x_k-l)\geq\epsilon\}|\min([f(\epsilon)]^h,[f(\epsilon)]^H).\end{aligned}$$

Hence $x\in S_u^q(\Delta_n^m)$.

Theorem 3.3. Let f be a bounded modulus function; then

$$S_u^q(\Delta_n^m)\subset w_1(\Delta_n^m,f,q,p,u).$$

Proof. Suppose that f is bounded. Let $\epsilon>0$ and let $\sum_1$ and $\sum_2$ be the sums introduced in the theorem 3.2. Since f is bounded, there exists an integer K such that $f(x)<k$, for all $x\geq 0$. Then

$$\begin{aligned}\frac{1}{n}\sum_{k=1}^{n}[f(q(u_k\Delta_n^m x_k-l))]^{p_k} &\leq \frac{1}{n}\Big(\sum_{1}[f(q(u_k\Delta_n^m x_k-l))]^{p_k}+\sum_{2}[f(q(u_k\Delta_n^m x_k-l))]^{p_k}\Big)\\ &\leq \frac{1}{n}\sum_{1}\max(K^h,K^H)+\frac{1}{n}\sum_{2}[f(\epsilon)]^{p_k}\\ &\leq \max(K^h,K^H)\frac{1}{n}|\{k\leq n: q(u_k\Delta_n^m x_k-l)\geq\epsilon\}|\\ &+ \max(f(\epsilon)^h,f(\epsilon)^H). \qquad (3.4)\end{aligned}$$

Hence $x\in w_1(\Delta_n^m,f,q,p,u)$.

Theorem 3.4. $S_u^q(\Delta_n^m)=w_1(\Delta_n^m,f,q,p,u)$ if and only if f is bounded.
Proof. Let f be bounded. By Theorems 3.2 and 3.3, we have $S_u^q(\Delta_n^m)=w_1(\Delta_n^m,f,q,p,u)$.
Conversely, suppose that f is unbounded. Then there exists a sequence (t_k) of positive numbers with $f(t_k)=k^2$, for $k=1,2,\cdots$. If we choose

$$u_i\Delta_n^m x_i=\begin{cases} t_k, & i=k^2, i=1,2,\cdots,\\ 0, & \text{otherwise}\end{cases}. \qquad (3.5)$$

Then we have

$$\frac{1}{n}|k\leq n: |u_k\Delta_n^m x_k|\geq\epsilon|\leq\frac{\sqrt{n}}{n} \qquad (3.6)$$

for all n and so $x\in S_u^q(\Delta_n^m)$ but $x\notin w_1(\Delta_n^m,f,q,p,u)$ for $X=\mathbb{C}, q(x)=|x|$ and $p_k=1$ for all $k\in\mathbb{N}$. This contradicts to $S_u^q(\Delta_n^m)=w_1(\Delta_n^m,f,q,p,u)$.

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