Compactification on soft fuzzy limit space

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Abstract: In this paper the concept of soft fuzzy filter on $X$, soft fuzzy prime filter on $X$, soft fuzzy ultrafilter on $X$, soft fuzzy minimal prime filter collections are introduced. The concept of soft fuzzy limit space $(X, l_t \tau)$ and the related properties are discussed. The functor $\iota$ from the collection of soft fuzzy filters $\mathfrak{F}(X)$ to the collection of filters on $X$ and the functor $\omega$ defined from the collection of filters on $X$ to the soft fuzzy filters $\mathfrak{F}(X)$ are defined and some of their properties are discussed. Soft fuzzy limit space $(X^*, lt \tau^*)$ defined from the existing soft fuzzy limit space $(X, l_t \tau)$. Also the process of compactification of soft fuzzy space $(X^*, lt \tau^*)$ is established.

Keywords: Soft fuzzy filter on $X(\mathfrak{F}(X))$; Soft fuzzy minimal prime filter collection $(\mathfrak{P}_m(\mathfrak{F}))$; Functor $\iota$ and $\omega$; Soft fuzzy limit space $(X, l_t \tau)$; Soft fuzzy limit space $(X^*, lt \tau^*)$.

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1 Introduction:

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [7]. Fuzzy sets have applications in many fields such as information [4] and control [5]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang [2] introduced and developed the concept of fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by Ismail U. Tiryaki [6]. Gunther Jager [3] discussed the Richardson compactification for fuzzy convergence spaces.

In this paper soft fuzzy filter $\mathfrak{F}$ on $X$, soft fuzzy prime filter, soft fuzzy minimal prime filter collection $\mathfrak{P}_m(\mathfrak{F})$ are introduced and studied. Some of their properties are discussed. Soft fuzzy limit space $(X, l_t \tau)$ is introduced. Soft fuzzy limit space $(X^*, lt \tau^*)$ is defined from the existing soft fuzzy limit space $(X, l_t \tau)$ has been established. Also the process of compactification of soft fuzzy limit space $(X^*, lt \tau^*)$ has been established.

2 Preliminaries:

Definition 2.1. \lA filter on a set $X$ is a set $F$ of subsets of $X$ which has the following properties:

(i) Every subset of $X$ which contains a set of $F$ belongs to $F$.
(ii) Every finite intersection of sets of $F$ belongs to $F$.
(iii) The empty set is not in $F$. 

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It follows from (i) and (ii) that every finite intersection of sets of \( F \) is non-empty.

**Definition:** 2.2. [1] An ultrafilter on a set \( X \) is a filter \( U \) such that there is no filter on \( X \) which is strictly finer than \( U \) (in other words, a maximal element in the ordered set of all filters on \( X \)).

**Definition:** 2.3. [2] A fuzzy subset in \( X \) is a function with domain \( X \) and value in \( I \), that is, an element of \( I^X \).

**Definition:** 2.4. [6] Let \( X \) be a set, \( \mu \) be a fuzzy subset of \( X \) and \( M \subseteq X \). Then, the pair \((\mu, M)\) will be called a soft fuzzy subset of \( X \). The set of all soft fuzzy subsets of \( X \) will be denoted by \( \text{SF}(X) \).

**Definition:** 2.5. [6] Let \( X \) be a non-empty set and the soft fuzzy sets \( A \) and \( B \) be in the form,

\[
A = \{(\mu, M)\}/\mu(x) \in I^X, \forall x \in X, M \subseteq X \}
\]
\[
B = \{(\lambda, N)\}/\lambda(x) \in I^X, \forall x \in X, N \subseteq X \}
\]

Then,

1. \( A \subseteq B \iff \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N. \)
2. \( A = B \iff \mu(x) = \lambda(x), \forall x \in X, M = N. \)
3. \( A' \iff 1 - \mu(x), \forall x \in X, X \mid M. \)
4. \( A \cap B \iff \mu(x) \land \lambda(x), \forall x \in X, M \cap N. \)
5. \( A \cup B \iff \mu(x) \lor \lambda(x), \forall x \in X, M \cup N. \)

**Definition:** 2.6. [6]

\( (0, \phi) = \{(\lambda, N)\}/\lambda = 0, N = \phi \)
\( (1, X) = \{(\lambda, N)\}/\lambda = 1, N = X \)

**Proposition:** 2.1. [6] If \((\mu_j, M_j) \in SF(X), j \in J\), then the family \(\{(\mu_j, M_j)\}, j \in J\) has a meet, that is the greatest lower bound, in \((SF(X), \sqsubseteq)\), denoted by

\[
\sqcap_{j \in J}(\mu_j, M_j) = (\sqcap_{j \in J} \mu_j, \sqcap_{j \in J} M_j)
\]

where

\[
(\sqcap_{j \in J} \mu_j)(x) = \sqcap_{j \in J} \mu_j(x) \text{ for all } x \in X.
\]

**Proposition:** 2.2. [6] If \( (\mu_j, M_j), j \in J \in SF(X) \), then the family \(\{(\mu_j, M_j)\}, j \in J\) has a join, that is least upper bound, in \((SF(X), \sqcup)\), denoted by

\[
\sqcup_{j \in J}(\mu_j, M_j) = (\sqcup_{j \in J} \mu_j, \sqcup_{j \in J} M_j)
\]

where

\[
(\sqcup_{j \in J} \mu_j)(x) = \sqcup_{j \in J} \mu_j(x) \text{ for all } x \in X.
\]

**Definition:** 2.7. [6] For \((\mu, M) \in SF(X)\) the soft fuzzy set \((\mu, M)' = (1 - \mu, X \setminus M)\) is called the complement of \((\mu, M)\).

**Definition:** 2.8. [6] Let \( x \in X \) and \( S \in I \) define \( x_s : X \rightarrow I \) by,

\[
x_s(z) = \begin{cases} 
s, & \text{if } z = x \\ 
0, & \text{otherwise}
\end{cases}
\]

Then the soft fuzzy set \((x_s, \{x\})\) is called the point of \( SF(X) \) with base \( x \) and value \( s \).

**Definition:** 2.9. [6] The soft fuzzy point \((x_r, \{x\}) \subseteq (\mu, M)\) is denoted by \((x_r, \{x\}) \in (\mu, M)\).
3 Soft fuzzy filter on $X$

**Definition: 3.1.** A collection $\mathcal{F} \subset SF(X)$ is called a soft fuzzy filter on $X$ iff

(i) If $\lambda = 0$ or $N = \phi \Rightarrow (\lambda, N) \not\in \mathcal{F} \neq (0, \phi)$.
(ii) If $(\lambda, N), (\mu, M) \in \mathcal{F} \Rightarrow (\lambda, N) \cap (\mu, M) \in \mathcal{F}$.
(iii) If $(1, X) \supseteq (\mu, M) \supseteq (\lambda, N) \in \mathcal{F} \Rightarrow (\mu, M) \in \mathcal{F}$

**Definition: 3.2.** A collection $\mathcal{B} \subset SF(X)$ is called soft fuzzy filter basis on $X$ iff

(i) If $\lambda = 0$ or $N = \phi \Rightarrow (\lambda, N) \not\in \mathcal{B} \neq (0, \phi)$.
(ii) If $(\lambda_1, N_1), (\lambda_2, N_2) \in \mathcal{B} \Rightarrow \exists (\lambda_3, N_3) \in \mathcal{B} \ni (\lambda_3, N_3) \subseteq (\lambda_1, N_1) \cap (\lambda_2, N_2)$.

**Definition: 3.3.** A soft fuzzy filter $\mathcal{F}$ on $X$ is called a soft fuzzy prime filter on $X$ iff $(\lambda_1, N_1) \cup (\lambda_2, N_2) \in \mathcal{F}$ with $(\lambda_1, N_1)$ or $(\lambda_2, N_2)$ is non empty such that $(\lambda_1, N_1) \in \mathcal{F}$ or $(\lambda_2, N_2) \in \mathcal{F}$.

**Notation: 3.1.** $(\lambda, N) = \{x : \lambda(x) > 0 \text{ and } x \in N\}$.

**Definition: 3.4.** Let $X$ be a non void set and $\alpha \in (0, 1], (\alpha_1(x), \{x\})$ represents the soft fuzzy point $(x, \alpha_1, \{x\})$ where $\alpha \in (0, 1]$.

$$\langle (\alpha_1(x), \{x\}) \rangle = \{(\lambda, N) \in SF(X) : (1, X) \supseteq (\lambda, N) \supseteq (x, \alpha, \{x\})\}$$

is the soft fuzzy point filters on $X$.

The set $\mathcal{F}(X) = \{\mathcal{F} : \mathcal{F} \text{ is a soft fuzzy filter on } X\}$ is ordered by inclusion. That is $\mathcal{F} \subset \mathcal{G}$. For $\mathcal{F} \in \mathcal{F}(X)$ the set $\mathcal{P}(\mathcal{F}) = \{\mathcal{G} \in \mathcal{F}(X) : \mathcal{G} \supset \mathcal{F}, \mathcal{G} \text{ is a soft fuzzy prime filter on } X\}$ is inductive and by zorn's lemma there exists a minimal element in $\mathcal{P}(\mathcal{F})$. The set of all minimal elements in $\mathcal{P}(\mathcal{F})$ is denoted by $\mathcal{P}_m(\mathcal{F})$.

**Proposition: 3.1.** The set $\mathcal{P}(\mathcal{F})$ is inductive in the sense that every decreasing chain of soft fuzzy filters on $X$ in $\mathcal{P}(\mathcal{F})$ has a lower bound.

**Definition: 3.5.** For a soft fuzzy filter $\mathcal{F}$ on $X$ we have $C(\mathcal{F}) = \bigwedge_{(\lambda, N) \in \mathcal{F}} \sum_{x \in X} \lambda(x)1_X \cap \bigvee_{(\lambda, N) \in \mathcal{F}} N$ is the characteristic value of $\mathcal{F}$ on $X$.

The connection between soft fuzzy filters on $X(\mathcal{F}(X))$ and filters on $X(F(X))$ is established by the mappings

$$\iota : \mathcal{F}(X) \rightarrow F(X) \quad \mathcal{F} \mapsto \iota(\mathcal{F}) = \{(\lambda, N) : (\lambda, N) \in \mathcal{F}\}$$

where $(\lambda, N)_o = \{x : \lambda(x) > 0 \text{ and } x \in N\}$ and

$$\omega : F(X) \rightarrow \mathcal{F}(X) \quad F \mapsto \omega(F) = \{(1_f, f) : f \in F\}_{(1, X)}$$

**Proposition: 3.2.** Let $\mathcal{F}$ be a soft fuzzy filter on $X$ and $F$ be a filter on $X$. The following hold

(i) $\iota(\omega(F)) = F$
(ii) $\omega(\iota(\mathcal{F})) \supseteq \mathcal{F}$
(iii) $\mathcal{F}$ is a soft fuzzy prime filter iff $\iota(\mathcal{F})$ is an ultrafilter.
(iv) $F$ is an ultrafilter iff $\omega(F)$ is a soft fuzzy prime filter.
Definition: 3.6. A filter $F$ on $X$ and a soft fuzzy filter $\mathcal{F}$ on $X$ are said to be compatible iff for all $f \in F$ and $(\mu, M) \in \mathcal{F}$, we have $(\mu, M) \cap (1_f, f) \neq (0, \phi)$. That is

$$(F, \mathcal{F}) = \langle \{(\mu, M) \cap (1_f, f) \mid (\mu, M) \in \mathcal{F}, f \in F\}\rangle_{(1, X)}$$

is a soft fuzzy filter on $X$.

Proposition: 3.3. Let $\mathcal{U}$ is an ultrafilter on $X$ and $\mathcal{F}$ is a soft fuzzy filter on $X$. Then $\mathcal{U}$ and $\mathcal{F}$ are compatible, if $\mathcal{U} \supset \iota(\mathcal{F})$.

Proposition: 3.4. Let $\mathcal{F}$ be a soft fuzzy filter on $X$. Then

$\mathcal{F}_m(\mathcal{F}) = \{\omega(\mathcal{U}) \cup \mathcal{F} : \mathcal{U} \supset \iota(\mathcal{F}), \mathcal{U}$ is ultrafilter}\.$

4 Soft fuzzy limit space and Soft fuzzy limit compact space

Definition: 4.1. Let $X$ be a non void set together with a mapping $\tau : X \to 2^{F(X)}$, $x \mapsto \tau(x)$ satisfying the axioms

(i) $(x) \in \tau(x), \forall x \in X$
(ii) $F \in \tau(x), G \supseteq F \Rightarrow G \in \tau(x)$
(iii) $U \in \tau(x), \forall U \supseteq F$ ultra $\Rightarrow F \in \tau(x)$.

Where $F(X)$ is collection of all filters on $X$. we define $S_\tau(F) = \{x \in X : F \in \tau(x)\}$.

Definition: 4.2. The pair $(X, lt_\tau)$ is called a soft fuzzy limit space iff

(i) For a soft fuzzy prime filter $\mathcal{F}$ on $X$ $lt_\tau(\mathcal{F}) = C(\mathcal{F}) \cap \{1_{S_\tau(\iota(\mathcal{F}))}, S_\tau(\iota(\mathcal{F}))\}$.
(ii) $\forall \mathcal{F} \in \mathcal{F}(X) : lt_\tau(\mathcal{F}) = \bigcap_{\mathcal{F} \in \mathcal{F}_m(\mathcal{F})} lt_\tau(\mathcal{F})$.

where $lt_\tau : \mathcal{F}(X) \to SF(X)$, $\mathcal{F} \mapsto lt_\tau(\mathcal{F})$.

Definition: 4.3. Let $(X, lt_\tau)$ be a soft fuzzy limit space. Then the soft fuzzy $lt_\tau$ closure of $(\lambda, N) \in SF(X)$ is defined by

$$SFlt_\tau - cl(\lambda, N) = \bigcup_{\mathcal{F} \in \mathcal{F}(X)} lt_\tau(\mathcal{F})$$

Definition: 4.4. A soft fuzzy set $(\lambda, N) \in SF(X)$ is called $SFlt_\tau$-closed in the soft fuzzy limit space iff whenever $(\lambda, N) \in \mathcal{F}$, $\mathcal{F}$ a soft fuzzy filter on $X$, then $lt_\tau(\mathcal{F}) \supseteq (\lambda, N)$. $(\lambda, N)$ is $SFlt_\tau$-closed iff $(\lambda, N) \supseteq SFlt_\tau - cl(\lambda, N)$.

Definition: 4.5. A soft fuzzy set $(\lambda, N)$ is called $SFlt_\tau$ - dense in the soft fuzzy limit space $(X, lt_\tau)$ iff

$$SFlt_\tau - cl(\lambda, N) = \{1, X\}$$

Definition: 4.6. Let $(X, lt_\tau)$ be a soft fuzzy limit space and $(\lambda, N) \subseteq (1, X)$. Then $\{X, lt_\tau\} \cap (\lambda, N)$ is a soft fuzzy limit space. $lt_\tau \mid (\lambda, N)$ is the soft fuzzy limit space induced by $lt_\tau$ and the pair $(X, lt_\tau) \mid (\lambda, N)$ is a soft fuzzy limit subspace of $(X, lt_\tau)$.

Definition: 4.7. A soft fuzzy limit space $(X, lt_\tau)$ is said to be soft fuzzy limit compact iff for every soft fuzzy prime filter $\mathcal{F} \in \mathcal{F}(X)$ we have $lt_\tau(\mathcal{F}) = C(\mathcal{F})$.
Definition: 4.8. Let $(X, t_X)$ be a non compact soft fuzzy limit space. $N(X) = \{ \mathfrak{F} \in \mathfrak{F}(X) \text{ soft fuzzy prime filter} : t_X(\mathfrak{F}) \sqsubseteq C(\mathfrak{F}) \}$ and define the equivalence relation $\sim$ on $\mathfrak{F}(X)$ by $\mathfrak{F} \sim \mathfrak{G}$ if $\iota(\mathfrak{F}) = \iota(\mathfrak{G})$. Let $[\mathfrak{F}] = \{ \mathfrak{G} \in \mathfrak{F}(X) : \mathfrak{F} \sim \mathfrak{G} \}$ and $D(X) = \{ ([\mathfrak{F}] : \mathfrak{F} \in N(X)) \}$. 

Definition: 4.9. Let $X^* = X \cup D(X)$ and for $[\mathfrak{F}] \in D(X)$ we define the characteristic value $C([\mathfrak{F}]) = (\bigvee_{(\lambda,N) \in \mathfrak{F}} \bigwedge_{x \in X} \lambda(x)1_{D(X)}, \{ ([\mathfrak{F}]) \})$.

Definition: 4.10. Let $X$ and $X^*$ be any two non empty sets and the soft fuzzy sets $(\lambda, N) \in SF(X)$ and $(\mu^*, M^*) \in SF(X^*)$. Then

(i) $(\lambda, N) \sqcap (\mu^*, M^*) = (\lambda(x) \wedge \mu^*(x), \forall x \in X \cap X^*, N \cap M^*)$.

(ii) $(\lambda, N) \sqcup (\mu^*, M^*) = (\lambda(x) \vee \mu^*(x), \forall x \in X \cup X^*, N \cup M^*)$.

Definition: 4.11. For $(\lambda, N) \sqsubseteq (1, X)$, $(\lambda, N)' = (\lambda', N')$ is a soft fuzzy set in $X^*$ given by $\lambda'(x) = \lambda(x)$ for $x \in X$ and $\lambda'([\mathfrak{F}]) = 0 \forall [\mathfrak{F}] \in D(X)$, $N' = N$ and

$$(\lambda, N)'^+ = \bigcup_{(\mathfrak{F}) \in D(X)} (C([\mathfrak{F}]) \cap (1_{[\mathfrak{F}]}, \{ ([\mathfrak{F}]) \}))$$

Therefore $(\lambda, N)' = (\lambda, N) \sqcup (1, X).$ By the mapping $\iota : SF(X) \to SF(X^*)$ we embed $SF(X)$ in $SF(X^*)$. That is a soft fuzzy set $(\lambda, N) \in SF(X)$ as a soft fuzzy set in $X^*$.

$\mathfrak{F}, \mathfrak{G}, \mathfrak{H}, \ldots \in \mathfrak{F}(X) \to \text{Soft fuzzy filter on } X,$

$\Phi, \Psi, \ldots \in \mathfrak{F}(X^*) \to \text{Soft fuzzy filter on } X^*.$

To a soft fuzzy filter $\mathfrak{F} \in \mathfrak{F}(X)$ we correspond a soft fuzzy filter $\mathfrak{F}^* \in \mathfrak{F}(X^*)$ by $\mathfrak{F}^* = \{ (\lambda, N)^* : (\lambda, N) \in \mathfrak{F} \}$ and to a soft fuzzy filter $\Phi \in \mathcal{F}(X^*)$ we correspond to a soft fuzzy filter $\Phi^* \in \mathfrak{F}(X)$ by $\Phi^* = \{ (\lambda, N) \sqsubseteq (1, X) : (\lambda, N)^* \in \Phi \}.$

Proposition: 4.1. Let $\mathfrak{F}, \mathfrak{G}$ be soft fuzzy prime filters on $X$, $t(\mathfrak{F}) = t(\mathfrak{G})$, $(\lambda, N) \sqsubseteq (\mu, M) \in \mathfrak{F}$ and $(\lambda, N) \sqcup (\mu, M) \in \mathfrak{G}$. Then $((\lambda, N) \in \mathfrak{F}$ and $(\lambda, N) \in \mathfrak{G}$ or $(\mu, M) \in \mathfrak{F}$ and $(\mu, M) \in \mathfrak{G}$).

Proposition: 4.2. For any soft fuzzy subset of $X$ we have

(i) $(0, \phi)^* = (0, \phi)$.

(ii) $(1, X)^* = (1, X^*)$.

(iii) $((\lambda, N) \sqcap (\mu, M))^* = (\lambda, N)^* \sqcap (\mu, M)^*$.

(iv) $((\lambda, N) \sqcup (\mu, M))^* = (\lambda, N)^* \sqcup (\mu, M)^*$.

(v) $(\lambda, N)^* \sqsubseteq (1, X) = (\lambda, N)$.

Proposition: 4.3. If $\Phi \in \mathfrak{F}(X^*)$ is a soft fuzzy prime filter then also $\Phi^*$ is a soft fuzzy prime filter in $X$.

Proposition: 4.4. Let $\mathfrak{F} \in \mathfrak{F}(X)$. Then $\langle \mathfrak{F}^* \rangle_{(1, X^*)} = \mathfrak{F}$.

Proposition: 4.5. Let $\Phi \in \mathfrak{F}(X^*)$. Then $\Phi^* \sqsubseteq \Phi$. If furthermore $\mathfrak{G} \in \mathfrak{F}(X)$ such that $\mathfrak{F}^* \sqsubseteq \Phi$ then $\mathfrak{G} \sqsubseteq \Phi^*$.

Proposition: 4.6. Let $\mathfrak{G} \in \mathfrak{F}(X)$. Then $\mathfrak{G} = \langle \Phi \in \mathfrak{F}(X^*) : (\mathfrak{F}) \subseteq \mathfrak{G} \exists (1, X^*) \rangle \{ \mathfrak{G} \} \}$.

Proposition: 4.7. For $\Phi \in \mathfrak{F}(X^*)$ we have that $C(\Phi) \sqsubseteq C(\Phi^*) \sqsubseteq \sqcup (D(X), D(X))$.

Proposition: 4.8. Let $\mathfrak{G} \in N(X)$ and $(0, \phi) \sqsubseteq (\alpha_1([\mathfrak{G}]), \{ ([\mathfrak{G}]) \}) \subseteq C([\mathfrak{G}])$. Then $\iota(\mathfrak{G}) \subseteq \iota((\alpha_1([\mathfrak{G}]), \{ ([\mathfrak{G}]) \}))$.

Proposition: 4.9. Let $\Phi, \Psi \in \mathfrak{F}(X^*)$. Then

(i) $\Phi \subseteq \Psi \Rightarrow C(\Phi) \sqsubseteq C(\Psi)$.

(ii) $\Phi \subseteq \Psi \Rightarrow \Phi^* \subseteq \Phi^*$.

(ii) $C(\Phi) = C(\Psi) \Rightarrow \iota(\Phi) = \iota(\Psi)$.  

5 Soft fuzzy limit space compactification

Let $X^*$ be a non void set together with the mapping $\tau^* : X^* \to 2^{F(X^*)}$, $x \mapsto \tau^*(x)$ satisfying the axioms

(i) $\langle x \rangle \in \tau^*(x)$, $\forall x \in X^*$.

(ii) $F \in \tau^*(x)$, $G \supseteq F \Rightarrow G \in \tau^*(x)$.

(iii) $\mathcal{U} \in \tau^*(x)$, $\forall \mathcal{U} \supseteq F \Rightarrow F \in \tau^*(x)$.

where $F(X^*)$ is a collection of all filters on $X^*$. We define $S_{\tau^*}(F) = \{ x \in X^* : F \in \tau^*(x) \}$.

Now let us define a soft fuzzy limit space on $X^*$. For a soft fuzzy prime filters $\Phi \in \mathfrak{S}(X^*)$ we have

$$lt_{\tau^*}(\Phi) = (lt_{\tau^*}(\Phi))' \sqcup (lt_{\tau^*}(\Phi))^+$$

where $(lt_{\tau^*}(\Phi))' = (lt_{\tau^*}(\tilde{\Phi})) \cap C(\Phi)$ and $(lt_{\tau^*}(\Phi))^+ = C(\Phi) \cap (1_{D(X)}, D(X))$.

**Proposition 5.1.** For a soft fuzzy limit non compact space $(X, lt_{\tau^*})$ we have that $(X^*, lt_{\tau^*})$ is also a soft fuzzy limit space.

**Proof.** (i) To prove the first condition of a soft fuzzy limit space. Let $\Phi$ is a soft fuzzy prime filter on $X^*$

$$lt_{\tau^*}(\Phi) = (C(\Phi) \cap lt_{\tau^*}(\tilde{\Phi})) \sqcup (C(\Phi) \cap (1_{D(X)}, D(X)))$$

$$= C(\Phi) \cap (lt_{\tau^*}(\tilde{\Phi}) \sqcup (1_{D(X)}, D(X)))$$

$$= C(\Phi) \cap (C(\tilde{\Phi}) \cap (1_{S_{\tau^*}(\tilde{\Phi})}, S_{\tau^*}(\tilde{\Phi}))) \sqcup (1_{D(X)}, D(X)))$$

$$= C(\Phi) \cap ((1_{S_{\tau^*}(\tilde{\Phi})}, S_{\tau^*}(\tilde{\Phi}))) \sqcup (1_{D(X)}, D(X))))$$

$$= C(\Phi) \cap (1_{S_{\tau^*}(\tilde{\Phi})}, S_{\tau^*}(\tilde{\Phi})))$$

(ii) Second condition follows by definition of $lt_{\tau^*}$. \hfill \Box

**Proposition 5.2.** For a soft fuzzy limit non compact space $(X, lt_{\tau^*})$ the soft fuzzy limit space $(X^*, lt_{\tau^*})$ is a soft fuzzy limit compact space.

**Proof.** Let $\Phi \in \mathfrak{S}(X^*)$ be a soft fuzzy prime filter on $X^*$. Then $\tilde{\Phi} \in \mathfrak{S}(X)$ be a soft fuzzy prime filter. From the hypothesis we have $lt_{\tau^*} \Phi = C(\Phi)$. Now

$$lt_{\tau^*} \Phi = (lt_{\tau^*} \Phi) \sqcup (lt_{\tau^*} \Phi)^+$$

$$(lt_{\tau^*} \Phi)' = lt_{\tau^*} \tilde{\Phi} \cap C(\Phi)$$

$$(lt_{\tau^*} \Phi)' = (C(\tilde{\Phi}) \cap (1_{S_{\tau^*}(\tilde{\Phi})}, S_{\tau^*}(\tilde{\Phi}))) \sqcup (1_{D(X)}, D(X)))$$

$$(lt_{\tau^*} \Phi)^+ = C(\Phi) \cap (1_{D(X)}, D(X))$$

$$lt_{\tau^*} \Phi = (C(\tilde{\Phi}) \cap (1_{S_{\tau^*}(\tilde{\Phi})}, S_{\tau^*}(\tilde{\Phi}))) \sqcup (1_{D(X)}, D(X)))$$

$$lt_{\tau^*} \Phi = C(\Phi) \cap (1_{D(X)}, D(X)))$$

$$lt_{\tau^*} \Phi = C(\Phi) (By \ Proposition \ 4.7)$$

Hence $(X^*, lt_{\tau^*})$ is a soft fuzzy limit compact space. \hfill \Box

**Proposition 5.3.** For a soft fuzzy limit non compact space $(X, lt_{\tau^*})$ we have $lt_{\tau^*} |_{\{1, X\}} = lt_{\tau^*}$.
Proof. Let $\mathcal{F} \in \mathfrak{F}(X)$ be a soft fuzzy prime filter. $\langle \mathcal{F} \rangle_{(1,X^*)}$ is a soft fuzzy prime filter.

$$lt_{\tau^*} |_{(1,X)} \mathcal{F} = lt_{\tau^*} \langle \mathcal{F} \rangle_{(1,X^*)} \cap (1,X)$$

$$= lt_{\tau} \langle \mathcal{F} \rangle_{(1,X^*)} \cap C(\langle \mathcal{F} \rangle_{(1,X^*)}) \cap (1,X)$$

$$= lt_{\tau} \mathcal{F} \cap C(\langle \mathcal{F} \rangle_{(1,X^*)}) \cap (1,X) \quad (\text{By Proposition - 4.4})$$

$$= lt_{\tau} \mathcal{F}$$

Hence $lt_{\tau^*} |_{(1,X)} = lt_{\tau}$. □

Proposition: 5.4. For a soft fuzzy limit non compact space $(X,lt_{\tau})$, $(1,X)$ is $lt_{\tau^*}$-dense in $(X^*,lt_{\tau^*})$.

Proof. Obviously $SFlt_{\tau^*} - d(1,X) \subseteq (1,X^*)$. To prove the reverse inclusion

$$SFlt_{\tau^*} - d(1,X) \supseteq \bigcup_{\mathfrak{F} \in \mathfrak{F}(X)} lt_{\tau^*} \langle \mathcal{F} \rangle_{(1,X^*)}$$

$$= \bigcup_{\mathcal{F} \in \mathfrak{F}(X)} lt_{\tau^*} \langle \mathcal{F} \rangle_{(1,X^*)}$$

$$= \bigcup_{\mathcal{F} \in \mathfrak{F}(X)} lt_{\tau^*} \langle \mathcal{F} \rangle_{(1,X^*)}$$

$$= SFlt_{\tau^*} - d(1,X^*) \supseteq (1,X^*) \quad (\text{By definition of } SFlt_{\tau^*} - d)$$

Hence $SFlt_{\tau^*} - d(1,X) = (1,X^*)$. □

Proposition: 5.5. For a soft fuzzy limit non compact space $(X,lt_{\tau})$ the soft fuzzy limit space $(X^*,lt_{\tau^*})$ is a compactification. Thus $(X^*,lt_{\tau^*})$ is the compactification of a soft fuzzy limit space $(X,lt_{\tau})$.

Proof. Proof is obtained from Property - 5.1, 5.2, 5.3 and 5.4. □

Proposition: 5.6. $(X^*,lt_{\tau^*})$ be the compactification of a soft fuzzy limit space $(X,lt_{\tau})$. we have for each filter $\mathcal{F} \in \mathfrak{F}(X)$ and $\langle \mathcal{F} \rangle_{(1,X^*)} \in \mathfrak{F}(X^*)$, $(lt_{\tau^*} \langle \mathcal{F} \rangle_{(1,X^*)}) \cap (1,X) = (lt_{\tau} \mathcal{F})^* \cap (1,X)$.

References


