American J. of Mathematics and Sciences
Vol. 3, No -1 ,(January 2014)

ISSN No: 2250-3102

Compactification on soft fuzzy limit space

V.Visalakshi, M.K.Uma and E.Roja

Department of Mathematics,
Sri Sarada College for Women,
Salem-636016,
Tamil Nadu, India.
e-mail : visalkumar_cbe@yahoo.co.in

Abstract: In this paper the concept of soft fuzzy filter on X, soft fuzzy prime filter on X, soft fuzzy ultrafilter on X, soft fuzzy minimal prime filter collections are introduced. The concept of soft fuzzy limit space (X, lt_τ) and the related properties are discussed. The functor ι from the collection of soft fuzzy filters $\mathfrak{F}(X)$ to the collection of filters on X and the functor ω defined from the collection of filters on X to the soft fuzzy filters $\mathfrak{F}(X)$ are defined and some of their properties are discussed. Soft fuzzy limit space (X^*, lt_{τ^*}) defined from the existing soft fuzzy limit space (X, lt_τ). Also the process of compactification of soft fuzzy space (X^*, lt_{τ^*}) is established.

Keywords: Soft fuzzy filter on $X(\mathfrak{F}(X))$; Soft fuzzy minimal prime filter collection $(\mathfrak{P}_{\mathfrak{m}}(\mathfrak{F}))$; Functor ι and ω; Soft fuzzy limit space (X, lt_τ); Soft fuzzy limit space (X^*, lt_{τ^*}).

MSC 2010.2000 Mathematics Subject Classification. 54A40, 03E72

1 Introduction:

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [7]. Fuzzy sets have applications in many fields such as information [4] and control [5]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Chang [2] introduced and developed the concept of fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by Ismail U.Tiryaki [6]. Gunther Jager [3] discussed the Richardson compactification for fuzzy convergence spaces.

In this paper soft fuzzy filter $\mathfrak{F}$ on X, soft fuzzy prime filter, soft fuzzy minimal prime filter collection $\mathfrak{P}_{\mathfrak{m}}(\mathfrak{F})$ are introduced and studied. Some of their properties are discussed. Soft fuzzy limit space (X, lt_τ) is introduced. Soft fuzzy limit space (X^*, lt_{τ^*}) is defined from the existing soft fuzzy limit space (X, lt_τ) has been established. Also the process of compactification of soft fuzzy limit space (X^*, lt_{τ^*}) has been established.

2 Preliminaries:

Definition: 2.1. *[1]*A filter on a set X is a set F of subsets of X which has the following properties:

(i) Every subset of X which contains a set of F belongs to F.

(ii) Every finite intersection of sets of F belongs to F.

(iii) The empty set is not in F.

It follows from (i) and (ii) that every finite intersection of sets of F is non empty.

Definition: 2.2. *[1]*An ultrafilter on a set X is a filter $\mathcal{U}$ such that there is no filter on X which is strictly finer than $\mathcal{U}$ (in other words, a maximal element in the ordered set of all filters on X).

Definition: 2.3. *[2]*A fuzzy subset in X is a function with domain X and value in I, that is, an element of I^X.

Definition: 2.4. *[6]*Let X be a set, μ be a fuzzy subset of X and M $\subseteq$ X. Then, the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by SF(X).

Definition: 2.5. *[6]*Let X be a non-empty set and the soft fuzzy sets A and B be in the form,

$$A = \{(\mu, M)/\mu(x) \in I^X, \forall x \in X, M \subseteq X\}$$
$$B = \{(\lambda, N)/\lambda(x) \in I^X, \forall x \in X, N \subseteq X\}$$

Then,

(1) $A \sqsubseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N.$

(2) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall x \in X, M = N.$

(3) $A' \Leftrightarrow 1 - \mu(x), \forall x \in X, X \mid M.$

(4) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X, M \cap N.$

(5) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X, M \cup N.$

Definition: 2.6. *[6]*
$(0, \phi)$= $\{(\lambda, N)/\lambda = 0, N = \phi\}$
$(1, X)$= $\{(\lambda, N)/\lambda = 1, N = X\}$

Proposition: 2.1. *[6]*If $(\mu_j, M_j) \in SF(X)$, $j \in J$, then the family $\{(\mu_j, M_j)|j \in J\}$ has a meet, that is the greatest lower bound, in $(SF(X), \sqsubseteq)$, denoted by

$$\sqcap_{j\in J}(\mu_j, M_j) = (\wedge_{j\in J}\mu_j, \cap_{j\in J}M_j)$$

where

$$(\wedge_{j\in J}\mu_j)(x) = \wedge_{j\in J}\mu_j(x) \text{ for all } x \in X.$$

Proposition: 2.2. *[6]*If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j)|j \in J\}$ has a join, that is least upper bound, in $(SF(X), \sqsubseteq)$, denoted by

$$\sqcup_{j\in J}(\mu_j, M_j) = (\vee_{j\in J}\mu_j, \cup_{j\in J}M_j)$$

where

$$(\vee_{j\in J}\mu_j)(x) = \vee_{j\in J}\mu_j(x) \text{ for all } x \in X.$$

Definition: 2.7. *[6]*For $(\mu, M) \in SF(X)$ the soft fuzzy set $(\mu, M)^{'} = (1-\mu, X\backslash M)$ is called the complement of (μ, M).

Definition: 2.8. *[6]*Let $x \in X$ and $S \in I$ define x_s: $X \to I$ by,

$$x_s(z) = \begin{cases} s, & \text{if z=x} \\ 0, & \text{otherwise} \end{cases}$$

Then the soft fuzzy set $(x_s, \{x\})$ is called the point of SF(X) with base x and value s.

Definition: 2.9. *[6]*The soft fuzzy point $(x_r, \{x\}) \sqsubseteq (\mu, M)$ is denoted by $(x_r, \{x\}) \in (\mu, M)$.

3 Soft fuzzy filter on X

Definition: 3.1. A collection $\mathfrak{F} \subset SF(X)$ is called a soft fuzzy filter on X iff

(i) If $\lambda = 0$ or $N = \phi \Rightarrow (\lambda, N) \notin \mathfrak{F} \neq (0, \phi)$.

(ii) If $(\lambda, N), (\mu, M) \in \mathfrak{F} \Rightarrow (\lambda, N) \sqcap (\mu, M) \in \mathfrak{F}$.

(iii) If $(1, X) \sqsupseteq (\mu, M) \sqsupseteq (\lambda, N) \in \mathfrak{F} \Rightarrow (\mu, M) \in \mathfrak{F}$

Definition: 3.2. A collection $\mathcal{B} \subset SF(X)$ is called soft fuzzy filter basis on X iff

(i) If $\lambda = 0$ or $N = \phi \Rightarrow (\lambda, N) \notin \mathcal{B} \neq (0, \phi)$.

(ii) If $(\lambda_1, N_1), (\lambda_2, N_2) \in \mathcal{B} \Rightarrow \exists (\lambda_3, N_3) \in \mathcal{B} \ni (\lambda_3, N_3) \sqsubseteq (\lambda_1, N_1) \sqcap (\lambda_2, N_2)$.

Definition: 3.3. A soft fuzzy filter $\mathfrak{F}$ on X is called a soft fuzzy prime filter on X iff $(\lambda_1, N_1) \sqcup (\lambda_2, N_2) \in \mathfrak{F}$ with $\overbrace{(\lambda_1, N_1)}$ or $\overbrace{(\lambda_2, N_2)}$ is non empty such that $(\lambda_1, N_1) \in \mathfrak{F}$ or $(\lambda_2, N_2) \in \mathfrak{F}$.

Notation: 3.1. $\overbrace{(\lambda, N)} = \{x : \lambda(x) > 0 \text{ and } x \in N\}$.

Definition: 3.4. Let X be a non void set and $\alpha \in (0, 1]$, $(\alpha 1_{\{x\}}, \{x\})$ represents the soft fuzzy point $(x_\alpha, \{x\})$ where $\alpha \in (0, 1]$.

$$\langle (\alpha 1_{\{x\}}, \{x\}) \rangle = \{(\lambda, N) \in SF(X) : (1, X) \sqsupseteq (\lambda, N) \sqsupseteq (x_\alpha, \{x\})\}$$

is the soft fuzzy point filters on X.

The set $\mathfrak{F}(X) = \{\mathfrak{F} : \mathfrak{F} \text{ is a soft fuzzy filter on X}\}$ is ordered by inclusion. That is $\mathfrak{F} \subset \mathfrak{G}$. For $\mathfrak{F} \in \mathfrak{F}(X)$ the set $\mathfrak{P}(\mathfrak{F}) = \{\mathfrak{G} \in \mathfrak{F}(X) : \mathfrak{G} \supset \mathfrak{F}, \mathfrak{G} \text{ is a soft fuzzy prime filter on X}\}$ is inductive and by zorn's lemma there exists a minimal element in $\mathfrak{P}(\mathfrak{F})$. The set of all minimal elements in $\mathfrak{P}(\mathfrak{F})$ is denoted by $\mathfrak{P}_{\mathfrak{m}}(\mathfrak{F})$

Proposition: 3.1. The set $\mathfrak{P}(\mathfrak{F})$ is inductive in the sense that every decreasing chain of soft fuzzy filters on X in $\mathfrak{P}(\mathfrak{F})$ has a lower bound.

Definition: 3.5. For a soft fuzzy filter $\mathfrak{F}$ on X we have $C(\mathfrak{F}) = (\bigwedge_{(\lambda,N)\in\mathfrak{F}} \bigvee_{x\in X} \lambda(x) 1_X, \bigcap_{(\lambda,N)\in\mathfrak{F}} N)$ is the characteristic value of $\mathfrak{F}$ on X.

The connection between soft fuzzy filters on $X(\mathfrak{F}(X))$ and filters on $X(F(X))$ is established by the mappings

$$\iota : \begin{cases} \mathfrak{F}(X) \to F(X) \\ \mathfrak{F} \mapsto \iota(\mathfrak{F}) = \{(\lambda, N)_\circ : (\lambda, N) \in \mathfrak{F}\} \end{cases}$$

where $(\lambda, N)_\circ = \{x : \lambda(x) > 0 \text{ or } x \in N\}$ and

$$\omega : \begin{cases} F(X) \to \mathfrak{F}(X) \\ F \mapsto \omega(F) = \langle \{(1_f, f)\} : f \in F \rangle_{(1,X)} \end{cases}$$

Proposition: 3.2. Let $\mathfrak{F}$ be a soft fuzzy filter on X and F be a filter on X. The following hold

(i) $\iota(\omega(\text{F})) = \text{F}$

(ii) $\omega(\iota(\mathfrak{F})) \sqsubseteq \mathfrak{F}$

(iii) $\mathfrak{F}$ is a soft fuzzy prime filter iff $\iota(\mathfrak{F})$ is an ultrafilter.

(iv) F is an ultrafilter iff $\omega(F)$ is a soft fuzzy prime filter.

Proof. Proof is clear ☐

Definition: 3.6. A filter F on X and a soft fuzzy filter $\mathfrak{F}$ on X are said to be compatible iff for all $f \in F$ and $(\mu, M) \in \mathfrak{F}$, we have $(\mu, M) \sqcap (1_f, f) \neq (0, \phi)$. That is

$$(F, \mathfrak{F}) = \langle \{(\mu, M) \sqcap (1_f, f) \mid (\mu, M) \in \mathfrak{F}, f \in F\} \rangle_{(1,X)}$$

is a soft fuzzy filter on X.

Proposition: 3.3. Let $\mathcal{U}$ is an ultrafilter on X and $\mathfrak{F}$ is a soft fuzzy filter on X. Then $\mathcal{U}$ and $\mathfrak{F}$ are compatible, if $\mathcal{U} \supset \iota(\mathfrak{F})$.

Proposition: 3.4. Let $\mathfrak{F}$ be a soft fuzzy filter on X. Then
$\mathfrak{P}_m(\mathfrak{F}) = \{\omega(\mathcal{U}) \cup \mathfrak{F} : \mathcal{U} \supseteq \iota(\mathfrak{F}), \mathcal{U} \text{ is ultrafilter}\}$.

4 Soft fuzzy limit space and Soft fuzzy limit compact space

Definition: 4.1. Let X be a non void set together with a mapping $\tau : X \to 2^{F(X)}$, $x \mapsto \tau(x)$ satisfying the axioms

(i) $\langle x \rangle \in \tau(x)$, $\forall x \in X$

(ii) $F \in \tau(x)$, $G \supseteq F \Rightarrow G \in \tau(x)$

(iii) $\mathcal{U} \in \tau(x)$, $\forall \mathcal{U} \supseteq F$ ultra $\Rightarrow F \in \tau(x)$.

Where F(X) is collection of all filters on X. we define $S_\tau(F) = \{x \in X : F \in \tau(x)\}$.

Definition: 4.2. The pair (X, lt_τ) is called a soft fuzzy limit space iff

(i) For a soft fuzzy prime filter $\mathfrak{F}$ on X $lt_\tau(\mathfrak{F}) = C(\mathfrak{F}) \sqcap (1_{S_\tau(\iota(\mathfrak{F}))}, S_\tau(\iota(\mathfrak{F})))$.

(ii) $\forall \mathfrak{F} \in \mathfrak{F}(X) : lt_\tau(\mathfrak{F}) = \sqcap_{\mathfrak{G} \in \mathfrak{P}_m(\mathfrak{F})} lt_\tau(\mathfrak{G})$.

where $lt_\tau : \mathfrak{F}(X) \to SF(X)$, $\mathfrak{F} \mapsto lt_\tau(\mathfrak{F})$.

Definition: 4.3. Let (X, lt_τ) be a soft fuzzy limit space. Then the soft fuzzy lt_τ closure of $(\lambda, N) \in SF(X)$ is defined by

$$SFlt_\tau - cl(\lambda, N) = \bigsqcup_{\substack{\mathfrak{F} \in \mathfrak{F}(X) \\ soft\ fuzzy\ prime\ filter \\ (\lambda, N) \in \mathfrak{F}}} lt_\tau \mathfrak{F}$$

Definition: 4.4. A soft fuzzy set $(\lambda, N) \in SF(X)$ is called $SFlt_\tau$-closed in the soft fuzzy limit space iff whenever $(\lambda, N) \in \mathfrak{F}$, $\mathfrak{F}$ a soft fuzzy filter on X, then $lt_\tau \mathfrak{F} \sqsubset (\lambda, N)$. (λ, N) is $SFlt_\tau$-closed iff $(\lambda, N) \sqsupset SFlt_\tau$-cl$(\lambda, N)$.

Definition: 4.5. A soft fuzzy set (λ, N) is called $SFlt_\tau - dense$ in the soft fuzzy limit space (X, lt_τ) iff $SFlt_\tau - cl(\lambda, N) = (1, X)$.

Definition: 4.6. Let (X, lt_τ) be a soft fuzzy limit space and $(\lambda, N) \sqsubset (1, X)$. Then $(X, lt_\tau) \mid_{(\lambda,N)}$ is a soft fuzzy limit space. $lt_\tau \mid_{(\lambda,N)}$ is the soft fuzzy limit space induced by lt_τ and the pair $(X, lt_\tau) \mid_{(\lambda,N)}$ is a soft fuzzy limit subspace of (X, lt_τ).

Definition: 4.7. A soft fuzzy limit space (X, lt_τ) is said to be soft fuzzy limit compact iff for every soft fuzzy prime filter $\mathfrak{F} \in \mathfrak{F}(X)$ we have $lt_\tau \mathfrak{F} = C(\mathfrak{F})$.

Definition: 4.8. Let (X, lt_τ) be a non compact soft fuzzy limit space. $N(X) = \{\mathfrak{F} \in \mathfrak{F}(X)$ soft fuzzy prime filter : $lt_\tau(\mathfrak{F}) \sqsubset C(\mathfrak{F})\}$ and define the equivalence relation $\backsim$ on $\mathfrak{F}(X)$ by $\mathfrak{F} \backsim \mathfrak{G}$ iff $\iota(\mathfrak{G}) = \iota(\mathfrak{F})$. Let $[\mathfrak{F}] = \{\mathfrak{G} \in \mathfrak{F}(X) : \mathfrak{G} \backsim \mathfrak{F}\}$ and $D(X) = \{[\mathfrak{F}] : \mathfrak{F} \in N(X)\}$.

Definition: 4.9. Let $X^* = X \cup D(X)$ and for $[\mathfrak{G}] \in D(X)$ we define the characteristic value $C([\mathfrak{G}]) = (\bigvee_{\mathfrak{F}\in[\mathfrak{G}]} \bigwedge_{(\lambda,N)\in\mathfrak{F}} \bigvee_{x\in X} \lambda(x) 1_{D(X)}, \{[\mathfrak{G}]\})$.

Definition: 4.10. Let X and X^* be any two non empty sets and the soft fuzzy sets $(\lambda, N) \in SF(X)$ and $(\mu^*, M^*) \in SF(X^*)$. Then

(i) $(\lambda, N)\overline{\sqcap}(\mu^*, M^*) = (\lambda(x) \wedge \mu^*(x), \forall x \in X \cap X^*, N \cap M^*)$.

(ii) $(\lambda, N)\underline{\sqcup}(\mu^*, M^*) = (\lambda(x) \vee \mu^*(x), \forall x \in X \cup X^*, N \cup M^*)$.

Definition: 4.11. For $(\lambda, N) \sqsubset (1, X)$, $(\lambda, N)' = (\lambda', N')$ is a soft fuzzy set in X^* given by $\lambda'(x) = \lambda(x)$ for $x \in X$ and $\lambda'([\mathfrak{G}]) = 0\ \forall [\mathfrak{G}] \in D(X)$, $N' = N$ and

$$(\lambda, N)^+ = \bigsqcup_{\substack{[\mathfrak{G}]\in D(X) \\ (\lambda,N)\in\mathfrak{F},\forall\mathfrak{F}\in[\mathfrak{G}]}} (C([\mathfrak{G}]) \sqcap (1_{\{[\mathfrak{G}]\}}, \{[\mathfrak{G}]\}))$$

Therefore $(\lambda, N)^* = (\lambda, N)'\underline{\sqcup}(\lambda, N)^+$. By the mapping $\prime : SF(X) \to SF(X^*)$ we embed $SF(X)$ in $SF(X^*)$. That is a soft fuzzy set $(\lambda, N) \in SF(X)$ as a soft fuzzy set in X^*.

$\mathfrak{F}, \mathfrak{G}, \mathfrak{H}, ... \in \mathfrak{F}(X) \to$ Soft fuzzy filter on X,
$\Phi, \Psi, ... \in \mathfrak{F}(X^*) \to$ Soft fuzzy filter on X^*.
To a soft fuzzy filter $\mathfrak{F} \in \mathfrak{F}(X)$ we correspond a soft fuzzy filter $\mathfrak{F}^* \in \mathfrak{F}(X^*)$ by $\mathfrak{F}^* = \langle\{(\lambda, N)^* : (\lambda, N) \in \mathfrak{F}\}\rangle_{(1,X^*)}$ and to a soft fuzzy filter $\Phi \in \mathcal{F}(X^*)$ we correspond to a soft fuzzy filter $\widetilde{\Phi} \in \mathfrak{F}(X)$ by $\widetilde{\Phi} = \{(\lambda, N) \sqsubset (1, X) : (\lambda, N)^* \in \Phi\}$.

Proposition: 4.1. Let $\mathfrak{F}, \mathfrak{G}$ be soft fuzzy prime filters on X, $\iota(\mathfrak{F}) = \iota(\mathfrak{G})$, $(\lambda, N) \sqcup (\mu, M) \in \mathfrak{F}$ and $(\lambda, N) \sqcup (\mu, M) \in \mathfrak{G}$. Then $((\lambda, N) \in \mathfrak{F}$ and $(\lambda, N) \in \mathfrak{G})$ or $((\mu, M) \in \mathfrak{F}$ and $(\mu, M) \in \mathfrak{G})$.

Proposition: 4.2. For any soft fuzzy subset of X we have

(i) $(0, \phi)^* = (0, \phi)$.

(ii) $(1, X)^* = (1, X^*)$.

(iii) $((\lambda, N) \sqcap (\mu, M))^* = (\lambda, N)^* \sqcap (\mu, M)^*$.

(iv) $((\lambda, N) \sqcup (\mu, M))^* = (\lambda, N)^* \sqcup (\mu, M)^*$.

(v) $(\lambda, N)^*\overline{\sqcap}(1, X) = (\lambda, N)$.

Proposition: 4.3. If $\Phi \in \mathfrak{F}(X^*)$ is a soft fuzzy prime filter then also $\widetilde{\Phi}$ is a soft fuzzy prime filter in X.

Proposition: 4.4. Let $\mathfrak{F} \in \mathfrak{F}(X)$. Then $\widetilde{\langle\mathfrak{F}\rangle_{(1,X^*)}} = \mathfrak{F}$.

Proposition: 4.5. Let $\Phi \in \mathfrak{F}(X^*)$. Then $\widetilde{\Phi}^* \subseteq \Phi$. If furthermore $\mathfrak{G} \in \mathfrak{F}(X)$ such that $\mathfrak{G}^* \subseteq \Phi$ then $\mathfrak{G} \subseteq \widetilde{\Phi}$.

Proposition: 4.6. Let $\mathfrak{F} \in \mathfrak{F}(X)$. Then $\mathfrak{P}_\mathfrak{m}(\mathfrak{F}) = \{\widetilde{\Phi} : \Phi \in \mathfrak{P}_\mathfrak{m}(\langle\mathfrak{F}\rangle_{(1,X^*)})\}$.

Proposition: 4.7. For $\Phi \in \mathfrak{F}(X^*)$ we have that $C(\Phi) \sqsubseteq C(\widetilde{\Phi})\underline{\sqcup}(1_{D(X)}, D(X))$.

Proposition: 4.8. *Let* $\mathfrak{G} \in N(X)$ *and* $(0, \phi) \sqsubseteq (\alpha 1_{\{[\mathfrak{G}]\}}, \{[\mathfrak{G}]\}) \sqsubseteq C([\mathfrak{G}])$. *Then* $\iota(\mathfrak{G}) \subseteq \iota(\widetilde{\langle\langle(\alpha 1_{\{[\mathfrak{G}]\}}, \{[\mathfrak{G}]\})\rangle\rangle})$.

Proposition: 4.9. Let $\Phi, \Psi \in \mathfrak{F}(X^*)$. Then

(i) $\Phi \subseteq \Psi \Rightarrow C(\Phi) \sqsupseteq C(\Psi)$.

(ii) $\Phi \subseteq \Psi \Rightarrow \widetilde{\Phi} \subseteq \widetilde{\Psi}$.

(ii) $\iota(\Phi) = \iota(\Psi) \Rightarrow \iota(\widetilde{\Phi}) = \iota(\widetilde{\Psi})$.

5 Soft fuzzy limit space compactification

Let X^* be a non void set together with the mapping $\tau^* : X^* \to 2^{F(X^*)}$, $x \mapsto \tau^*(x)$ satisfying the axioms

(i) $\langle x \rangle \in \tau^*(x)$, $\forall x \in X^*$.

(ii) $F \in \tau^*(x), G \supseteq F \Rightarrow G \in \tau^*(x)$.

(iii) $\mathcal{U} \in \tau^*(x)$, $\forall \mathcal{U} \supseteq F$ ultra $\Rightarrow F \in \tau^*(x)$.

where $F(X^*)$ is a collection of all filters on X^*. We define $S_{\tau^*}(F) = \{x \in X^* : F \in \tau^*(x)\}$.

Now let us define a soft fuzzy limit space on X^*. For a soft fuzzy prime filters $\Phi \in \mathfrak{F}(X^*)$ we have

$$lt_{\tau^*}(\Phi) = (lt_{\tau^*}(\Phi))' \sqcup (lt_{\tau^*}(\Phi))^+$$

where $(lt_{\tau^*}(\Phi))' = (lt_\tau \widetilde{\Phi}) \overline{\sqcap} C(\Phi)$ and $(lt_{\tau^*}\Phi)^+ = C(\Phi)\overline{\sqcap}(1_{D(X)}, D(X))$.

Proposition: 5.1. For a soft fuzzy limit non compact space (X, lt_τ) we have that (X^*, lt_{τ^*}) is also a soft fuzzy limit space.

Proof. (i) To prove the first condition of a soft fuzzy limit space. Let Φ is a soft fuzzy prime filter on X^*

$$\begin{aligned}
lt_{\tau^*}(\Phi) &= (C(\Phi)\overline{\sqcap} lt_\tau(\widetilde{\Phi})) \sqcup (C(\Phi)\overline{\sqcap}(1_{D(X)}, D(X))) \\
&= C(\Phi)\overline{\sqcap}(lt_\tau(\widetilde{\Phi}) \sqcup (1_{D(X)}, D(X))) \\
&= C(\Phi)\overline{\sqcap}(C(\widetilde{\Phi}) \sqcap (1_{S_\tau(\iota(\widetilde{\Phi}))}, S_\tau(\iota(\widetilde{\Phi}))) \sqcup (1_{D(X)}, D(X))) \\
&= C(\Phi)\overline{\sqcap}((C(\widetilde{\Phi}) \sqcup (1_{D(X)}, D(X))) \sqcap ((1_{S_\tau(\iota(\widetilde{\Phi}))}, S_\tau(\iota(\widetilde{\Phi}))) \sqcup (1_{D(X)}, D(X)))) \\
&= C(\Phi) \sqcap (1_{S_{\tau^*}(\iota(\Phi))}, S_{\tau^*}(\iota(\Phi)))
\end{aligned}$$

(ii) Second condition follows by definition of lt_{τ^*}.

□

Proposition: 5.2. For a soft fuzzy limit non compact space (X, lt_τ) the soft fuzzy limit space (X^*, lt_{τ^*}) is a soft fuzzy limit compact space.

Proof. Let $\Phi \in \mathfrak{F}(X^*)$ be a soft fuzzy prime filter on X^*. Then $\widetilde{\Phi} \in \mathfrak{F}(X)$ be a soft fuzzy prime filter. From the hypothesis we have $lt_\tau \widetilde{\Phi} = C(\widetilde{\Phi})$. Now

$$\begin{aligned}
lt_{\tau^*}\Phi &= (lt_{\tau^*}\Phi)' \sqcup (lt_{\tau^*}\Phi)^+ \\
(lt_{\tau^*}\Phi)' &= lt_\tau \widetilde{\Phi} \overline{\sqcap} C(\Phi) \\
(lt_{\tau^*}\Phi)' &= (C(\widetilde{\Phi}) \sqcap (1_{S_\tau(\iota(\widetilde{\Phi}))}, S_\tau(\iota(\widetilde{\Phi}))))\overline{\sqcap} C(\Phi) \\
(lt_{\tau^*}\Phi)^+ &= C(\Phi)\overline{\sqcap}(1_{D(X)}, D(X)) \\
lt_{\tau^*}\Phi &= (C(\widetilde{\Phi}) \sqcap (1_{S_\tau(\iota(\widetilde{\Phi}))}, S_\tau(\iota(\widetilde{\Phi})))\overline{\sqcap} C(\Phi)) \sqcup (C(\Phi)\overline{\sqcap}(1_{D(X)}, D(X))) \\
lt_{\tau^*}\Phi &= C(\Phi)\overline{\sqcap}(C(\widetilde{\Phi}) \sqcup (1_{D(X)}, D(X))) \\
lt_{\tau^*}\Phi &= C(\Phi) \ (By\ Proposition - 4.7)
\end{aligned}$$

Hence (X^*, lt_{τ^*}) is a soft fuzzy limit compact space. □

Proposition: 5.3. For a soft fuzzy limit non compact space (X, lt_τ) we have $lt_{\tau^*} |_{(1,X)} = lt_\tau$.

Proof. Let $\mathfrak{F} \in \mathfrak{F}(X)$ be a soft fuzzy prime filter. $\langle \mathfrak{F} \rangle_{(1,X^*)}$ is a soft fuzzy prime filter.

$$\begin{aligned} lt_{\tau^*} \mid_{(1,X)} \mathfrak{F} &= lt_{\tau^*} \langle \mathfrak{F} \rangle_{(1,X^*)} \overline{\sqcap}(1,X) \\ &= lt_{\tau} \widetilde{\langle \mathfrak{F} \rangle_{(1,X^*)}} \overline{\sqcap} C(\langle \mathfrak{F} \rangle_{(1,X^*)}) \overline{\sqcap}(1,X) \\ &= lt_{\tau} \mathfrak{F} \overline{\sqcap} C(\langle \mathfrak{F} \rangle_{(1,X^*)}) \overline{\sqcap}(1,X) \ (By\ Propostion - 4.4) \\ &= lt_{\tau} \mathfrak{F} \end{aligned}$$

Hence $lt_{\tau^*} \mid_{(1,X)} = lt_{\tau}$. □

Proposition: 5.4. For a soft fuzzy limit non compact space (X, lt_{τ}), $(1,X)$ is lt_{τ^*}-dense in (X^*, lt_{τ^*}).

Proof. Obviously $SFlt_{\tau^*} - cl(1,X) \sqsubseteq (1,X^*)$. To prove the reverse inclusion

$$\begin{aligned} SFlt_{\tau^*} - cl(1,X) &= \bigsqcup_{\substack{\mathfrak{F} \in \mathfrak{F}(X) \\ Soft\ fuzzy\ prime\ filter \\ (1,X) \in \mathfrak{F}}} lt_{\tau^*}(\langle \mathfrak{F} \rangle_{(1,X^*)}) \\ &= \bigsqcup_{\substack{\langle \mathfrak{F} \rangle_{(1,X^*)} \in \mathfrak{F}(X^*) \\ Soft\ fuzzy\ prime\ filter \\ (1,X^*) \in \langle \mathfrak{F} \rangle_{(1,X^*)}}} lt_{\tau^*}(\langle \mathfrak{F} \rangle_{(1,X^*)}) \\ &= SFlt_{\tau^*} - cl(1,X^*) \\ &\sqsupseteq (1,X^*) \ (By\ definition\ of\ SFlt_{\tau^*} - cl) \end{aligned}$$

Hence $SFlt_{\tau^*} - cl(1,X) = (1,X^*)$. □

Proposition: 5.5. For a soft fuzzy limit non compact space (X, lt_{τ}) the soft fuzzy limit space (X^*, lt_{τ^*}) is a compactification. Thus (X^*, lt_{τ^*}) is the compactification of a soft fuzzy limit space (X, lt_{τ}).

Proof. Proof is obtained from Property - 5.1, 5.2, 5.3 and 5.4. □

Proposition: 5.6. (X^*, lt_{τ^*}) be the compactification of a soft fuzzy limit space (X, lt_{τ}). we have for each filter $\mathfrak{F} \in \mathfrak{F}(X)$ and $\langle \mathfrak{F} \rangle_{(1,X^*)} \in \mathfrak{F}(X^*)$, $(lt_{\tau^*} \langle \mathfrak{F} \rangle_{(1,X^*)}) \overline{\sqcap}(1,X) = (lt_{\tau} \mathfrak{F})^* \overline{\sqcap}(1,X)$.

References

[1] N. Bourbaki, Elements of Mathematics General Topology, (1971) Springer-Verlag Berlin Heidelberg Newyork London Paris Tokyo.

[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1968),pp. 182-190.

[3] Gunther Jager, The Richardson compactificton for fuzzy convergence space, Fuzzy sets and systems, 92(1997), pp. 349-355.

[4] P. Smets, The degree of belief in a fuzzy event, Information Sciences, 25(1981), 1-19.

[5] M. Sugeno, An introductory Survey of fuzzy control, Information Sciences, 36(1985), 59-83.

[6] Ismail U. Tiryaki, Fuzzy sets over the poset I, Hacettepe. Journal of Mathematics and Statistics, Volume 37(2) (2008), 143-166.

[7] L. A. Zadeh, Fuzzy sets, Inform and control 8(1965) 338-353.