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On qαI- Irresolute Mappings

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Abstract

In the present paper the concept of qα$\mathcal{I}$-Irresolute mappings have been introduced and studied.

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1. Preliminaries

In 1965 Njastad [9] introduced the concept of α open sets in topology. A subset A of a topological space (X, τ) is said to be α open if A ⊂ int(Cl(int(A))). Every open set is α open but the converse may not be true. In 1963 Kelly [5] introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2 [5]. The study of quasi open sets in bitopological spaces was initiated by Datta [1] in 1971. In a bitopological space (X, τ_1, τ_2) a set A of X is said to be quasi open [1] if it is a union of a τ_1-open set and a τ_2-open set. Complement of a quasi open set is termed quasi closed. Every τ_1-open (resp. τ_2-open) set is quasi open but the converse may not be true. Any union of quasi open sets of X is quasi open in X. The intersection of all quasi closed sets which contains A is called quasi closure of A [7]. It is denoted by qcl(A). The union of quasi open subsets of A is called quasi interior of A. It is denoted by qInt(A) [7]. In 1985, Thakur and Paik [10] introduced the concept of quasi α open sets in bitopological spaces. A set A in a bitopological space (X, τ_1, τ_2) is called quasi α open [10] if it is a union of a $\tau_{1\alpha}$- open set and a $\tau_{2\alpha}$- open set. Complement of a quasi α open set is called quasi α closed. Every $\tau_{1\alpha}$- open ($\tau_{2\alpha}$- open, quasi open) set is quasi α open but the converse may not be true. Any union of quasi α open sets of X is a quasi α open set in X. The intersection of all quasi α closed sets which contains A is called quasi α closure of A. It is denoted by qαcl(A). The union of quasi α open subsets of A is called quasi α- interior of A. It is denoted by qαInt(A)[10]. Further, in 1980 Maheshwari and Thakur [8] introduced α-irresolute mappings. A mapping f: (X,τ) → (Y,σ) is called a α- irresolute if $f^{-1}(V)$ is a α- open set in X for every α- open set V of Y [8].

The concept of ideal topological spaces was initiated Kuratowski [6] and Vaidyanathaswamy [11]. An Ideal $\mathcal{I}$ on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: i) A $\in \mathcal{I}$ and B ⊂ A ⇒ B $\in \mathcal{I}$ and ii) A $\in \mathcal{I}$ and B $\in \mathcal{I}$ ⇒ A∪B $\in \mathcal{I}$ If $\mathcal{P}(X)$ is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*:\mathcal{P}(X) \rightarrow \mathcal{P}(X)$ called the local function [2] of A with respect to τ and $\mathcal{I}$ and is defined as follows:

$A^*(\tau, I) = \{x \in X \mid U \cap A \notin I, \forall U \in \tau(x)\}$, where $\tau(x) = U \in \tau \mid x \in U\}$.

Given an ideal bitopological space (X,τ_1,τ_2,I) the quasi local function [3] of A with respect to τ_1, τ_2 and I denoted by A^*_q (τ_1,τ_2,I) (in short A^*_q) is defined as follows:

$$A^*_q(\tau_1,\tau_2,I) = \{x \in X \mid U \cap A \notin I, \forall \text{ quasi open set } U \text{ containing } x\}.$$

A subset A of an ideal bitopological space (X, τ_1, τ_2) is said to be qI- open [3] if $A \subset \text{qInt } A^*_q$. A mapping f: $(X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is called qI-continuous [3] if $f^{-1}(V)$ is qI-open in X for every quasi open set V of Y. Recently the authors of this paper [4] defined qαI- open sets and qαI- continuous mappings in ideal bitopological spaces.

Definition1.1.[4] Given an ideal bitopological space (X,τ_1,τ_2,I) the quasi α- local mapping of A with respect to τ_1, τ_2 and I denoted by $A^*_{q\alpha}(\tau_1,\tau_2,I)$ (more generally as $A^*_{q\alpha}$) is defined as $A^*_{q\alpha}(\tau_1,\tau_2,I) = \{x \in X \mid U \cap A \notin I, \forall$ quasi α open set U containing x$\}$

Definition1.2. [4] A subset A of an ideal bitopological space (X,τ_1,τ_2,I) is qαI- open if $A \subset \text{q}\alpha\text{Int}(A^*_{q\alpha})$ and qαI-closed if its complement is qαI- open.

Remark1.1. [4] Every qI- open set is qαI- open but the converse is not true

Remark1.2. [4] The concepts of qαI- open sets and quasi α- open sets are independent.

Definition1.3.[4] A mapping f: $(X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is called a qαI- continuous if $f^{-1}(V)$ is a qαI- open set in X for every quasi open set V of Y

Remark1.3.[4] Every qI- continuous mapping is qαI- continuous but the converse is not true

Definition1.4.[4] In an ideal bitopological space (X,τ_1,τ_2,I) the quasi *- α closure of A of X denoted by $\text{q}\alpha\text{cl}^*(A)$ is defined by $\text{q}\alpha\text{cl}^*(A) = A \cup A^*_{q\alpha}$

Definition1.5.[4] A subset A of an ideal bitopological space (X,τ_1,τ_2,I) is said to be a qαI- neighbourhood of a point $x \in X$ if $\exists$ a qαI- open set O in X, such that $x \in O \subset A$

Definition1.6.[4] Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,I) and $x \in X$. Then x is called a qαI-interior point of A if $\exists$ V a qαI- open set in X such that $x \in V \subset A$.
The set of all qαI- interior points of A is called the qαI- interior of A and is denoted by qαIInt(A).

Definition1.7.[4] Let A be a subset of an ideal bitopological space (X,τ_1,τ_2,I) and $x \in X$. Then x is called a qαI-cluster point of A, if $V \cap A \neq \emptyset$. for every qαI- open set V in X. The set of all qαI-cluster points of A denoted by qαIcl(A) is called the qαI-closure of A .

2. qαI- Irresolute Mappings

Definition 2.1. A mapping f: $(X,\tau_1,\tau_2,I) \to (Y,\sigma_1,\sigma_2)$ is called qαI- irresolute if $f^{-1}(V)$ is a qαI- open set in X for every quasi α open set V of Y.

Remark 2.1. Every qαI- irresolute mapping is qαI- continuous but the converse may not true. For,

Example 2.1. Let X = {a, b, c} and $\mathcal{I}$ = {ϕ, {a}} be an ideal on X. Let τ_1 = {X, ϕ, {c}}, τ_2 = {X, ϕ, {a, b}}, σ_1 = {X, ϕ, {b}}and σ_2 = { ϕ, X} be topologies on X. Then the identity mapping f: $(X,\tau_1,\tau_2,\mathcal{I}) \rightarrow (X,\sigma_1,\sigma_2)$ is qα$\mathcal{I}$- continuous but not qα$\mathcal{I}$- irresolute.

Theorem 2.1. Let f: $(X,\tau_1,\tau_2,\mathcal{I}) \rightarrow (Y,\sigma_1,\sigma_2)$ be a mapping, then the following statements are equivalent:

- **(a)** f is qα$\mathcal{I}$- irresolute.
- **(b)** $f^{-1}(V)$ is qα$\mathcal{I}$- closed set in X for every quasi α- closed set V of Y
- **(c)** for each x ∈ X and every quasi α open set V of Y containing f(x), ∃ a qα$\mathcal{I}$- open set W of X containing x such that f(W) ⊂ V.
- **(d)** for each x ∈ X and every quasi α open set V of Y containing f(x), $f^{-1}(V)^*_{q\alpha}$ is a qα$\mathcal{I}$- neighbourhood of X.

Proof: (a) ⇔ (b). Obvious.

(a) ⇒ (c). Let x ∈ X and V be a quasi α open set of Y containing f(x). Since f is qα$\mathcal{I}$- irresolute, $f^{-1}(V)$ is a qα$\mathcal{I}$- open set. Put W = $f^{-1}(V)$, then x ∈ W. Hence f(W) ⊂ V.

(c) ⇒ (a). Let A be a quasi α open set in Y. If $f^{-1}(A) = \emptyset$, then $f^{-1}(A)$ is clearly a qα$\mathcal{I}$- open set. Assume that $f^{-1}(A) \neq \emptyset$ and x ∈ $f^{-1}(A)$, then f(x) ∈ A ⇒ ∃ a qα$\mathcal{I}$- open set W containing x such that f (W) ⊂ A. Thus, W ⊂ $f^{-1}(A)$. Since W is qα$\mathcal{I}$- open, x ∈ W ⊂ qαInt($W^*_{q\alpha}$) ⊂ qαInt($f^{-1}(A)^*_{q\alpha}$) and so $f^{-1}(A)$ ⊂ qαInt($f^{-1}(A)^*_{q\alpha}$). Hence $f^{-1}(A)$ is a qα$\mathcal{I}$- open set and so f is qα$\mathcal{I}$- irresolute.

(c) ⇒(d). Let x ∈ X and V be a quasi α open set of Y containing f(x) then ∃ a qα$\mathcal{I}$- open set W containing x such that f(W) ⊂ V Therefore W ⊂ $f^{-1}(V)$. Since W is a qα$\mathcal{I}$- open set, x ∈ W ⊂ qαInt($W^*_{q\alpha}$)) ⊂ qαInt($f^{-1}(V)^*_{q\alpha}$) ⊂ ($f^{-1}(V)^*_{q\alpha}$). Hence $f^{-1}(V)^*_{q\alpha}$) is a qα$\mathcal{I}$- neighbourhood of x.

(d) ⇒(c). Obvious.

Definition2.2. A mapping f**:** $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ is said to be :

- **(a)** qα$\mathcal{I}$- α open if f(U) is a qα$\mathcal{I}$- open set of Y for every quasi α open set U of X.
- **(b)** qα$\mathcal{I}$- α closed if f(U) is a qα$\mathcal{I}$- closed set of Y for every quasi α closed set U of X.

Theorem 2.2. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ be a mapping. Then the following statements are equivalent:

- **(a)** f is qα$\mathcal{I}$- α open
- **(b)** f(qαInt(U)) ⊂ qα$\mathcal{I}$Int(f(U) for each subset U of X.
- **(c)** qαInt($f^{-1}(V)$) ⊂ f^{-1}(qα$\mathcal{I}$Int(V)) for each subset V of Y.

Proof: (a) ⇒ (b). Let U be any subset of X. Then qαInt(U) is a quasi α open set of X. Then f(qαInt(U)) is a qα$\mathcal{I}$- open set of Y. Since f(qαInt(U)) ⊂ f(U), f(qαInt(U)) = qα$\mathcal{I}$Int(f(qαInt(U)) ⊂ qα$\mathcal{I}$Int(f(U).

(b) ⇒ (c). Let V be any subset of Y. Obviously $f^{-1}(V)$ is a subset of X. Therefore by (b), f(qαInt($f^{-1}(V)$)) ⊂ qα$\mathcal{I}$Int(f($f^{-1}(V)$)) ⊂ qα$\mathcal{I}$Int(V)). Hence, qαInt($f^{-1}(V)$) ⊂ f^{-1}(f(qαInt($f^{-1}(V)$))) ⊂ f^{-1}(qα$\mathcal{I}$Int(V))

(c) ⇒ (a). Let V be any quasi α open set of X. Then qαInt(V) = V and f(V) is a subset of Y. So V = qαInt(V) ⊂ qαInt(f^{-1}(f(V))) ⊂ f^{-1}(qα$\mathcal{I}$Int(f(V))). Then f(V) ⊂ f(f^{-1}(qα$\mathcal{I}$Int(f(V))) ⊂ qα$\mathcal{I}$Int(f(V) and qα$\mathcal{I}$Int(f(V) ⊂ f(V). Hence, f(V) is a qα$\mathcal{I}$-open set of Y and f is qα$\mathcal{I}$-open.

Theorem 2.3. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ be a qα$\mathcal{I}$- α open mapping. If V is a subset of Y and U is a quasi α closed subset of X containing $f^{-1}(V)$, then there exists a qα$\mathcal{I}$- closed set F of Y containing V such that $f^{-1}(F)$ ⊂ U.

Proof: Let V be any subset of Y and U a quasi α closed subset of X containing $f^{-1}(V)$, and let F = Y\(f(X\V)). Then f(X\V) ⊂ f(f^{-1}(X\V)) ⊂ (X\V) and X\U is a quasi α open set of X. Since f is qα$\mathcal{I}$- α open, f(X\U) is a qα$\mathcal{I}$- open set of Y. Hence F is a quasi α closed subset of Y and $f^{-1}(F)$ = f^{-1}(Y\(f (X\U)) ⊂ U.

Theorem 2.4. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ is qα$\mathcal{I}$- α closed if and only if qα$\mathcal{I}$cl(f(V) ⊂ f(qαcl(V) for each subset V of X.

Proof:
Necessity Let f be a q$\alpha\mathcal{I}$- α closed mapping and V be any subset of X. Then f(V) $\subset$ f(qαcl(V) and f(qαcl(V) is a q$\alpha\mathcal{I}$- closed set of Y. Thus, q$\alpha\mathcal{I}$cl(f(V)) $\subset$ q$\alpha\mathcal{I}$cl(f(qαcl(V)) = f(qαcl(V).
Sufficiency Let V be a quasi α closed set of X. Then by hypothesis f(V) $\subset$ q$\alpha\mathcal{I}$cl(f(V)) $\subset$ f(qαcl(V) = f(V) . And so, f(V) is a q$\alpha\mathcal{I}$- closed subset of Y. Hence, f is q$\alpha\mathcal{I}$- α closed.

Theorem 2.5. A mapping f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ is q$\alpha\mathcal{I}$- α closed if and only if f^{-1}(q$\alpha\mathcal{I}$cl(V)) $\subset$ qαcl(f^{-1}(V)) for each subset V of Y.

Proof: Obvious.

Theorem 2.6. Let f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2,\mathcal{I})$ be a q$\alpha\mathcal{I}$- α closed mapping. If V is a subset of Y and U is a quasi α open subset of X containing f^{-1}(V), then there exists a q$\alpha\mathcal{I}$- open set F of Y containing V such that $f^{-1}(F) \subset U$.

Proof: Obvious.

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