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FUZZY PAIRWISE GENERALIZED ρ-CLOSED SETS WHERE ρ ∈{ α,α*, α**} IN FUZZY BITOPOLOGICAL SPACES

Bhuvaneswari Balasubramanian[1] N.Chandramathi[2] and S.Bharathi[3]

[1] Department of Mathematics, Mother Teresa Women's University Kodaikanal, Tamilnadu, India.
Email:drkbmaths@gmail.com
[2] Department of Mathematics, Hindusthan college of Engineering and Technology,
Coimbatore-32, Tamilnadu, India
Email:mathi.chandra303@gmail.com
[3]Department of Mathematics, Angel college of Engineering and Technology,Tirupur (Dt),
Tamilnadu, India
Email:bharathikamesh6@gmail.com

Abstract

In this paper we introduce and study the concept of (τ_i, τ_j) -fuzzy generalized ρ-closed sets and studied some theorems based on this concept.

Keywords: (τ_i, τ_j) -fuzzy generalized ρ-closed sets where ρ∈{α,α*,α**}

1. INTRODUCTION

The fundamental concept of fuzzy sets was introduced by Zedeh in his classical paper[10]. Thereafter many investigations have been carried out in the general theoretical field and also in different application are as based in this concept. Chang
[4] used the concept of fuzzy sets to introduce fuzzy topological spaces and several other authors continued the investigation of such spaces .Devi et al [5] introduced fuzzy generalized α -closed sets and investigated its applications .

In this paper first we introduce (τ_i, τ_j) fuzzy generalized ρ -closed sets where ρ∈{α,α*,α**}in Section 3 and studied some of its applications.

2. PRELIMINARIES

Let X be a non empty set and I=[0,1]. A fuzzy set in X is a mapping from X into I. The null fuzzy set 0 is the mapping from X into I which assumes only the value 0 and fuzzy set 1 is a mapping from X into I which takes the value 1 only. The union $\cup A_\alpha$ (resp. intersection $\cap A_\alpha$) of a family { A_α : α ∈ ∧} of fuzzy sets of X is defined to be the mapping sup A_α (resp. Inf A_α) . A fuzzy set A of X contained in a fuzzy set B of X is denoted by A ≤ B if and only if A(x) ≤ B(x) for each x . The complement A^c of a fuzzy set A of X is 1-A defined by (1-A)(x) , for each x∈ X . A fuzzy point x_β in X is a fuzzy set in X defined by
$x_\beta(y)$ ={β(β∈(0,1]; for y = x (y ∈X)
{ 0 ; other wise
x and β are respectively, called the support and value of x_β . A fuzzy point $x_\beta \in A$ if and only if β ≤ A(x). A fuzzy set A is the union of all fuzzy points which belongs to A.

Definition 2.1[5] Let (X , τ) be a fuzzy topological space. A fuzzy set λ in X is called (i) fuzzy generalized α-closed (in short fgαc)⇔ αCl (λ) ≤ μ whenever λ ≤ μ and μ is a fuzzy α-open set.
(ii) fuzzy generalized α*-closed (in short fgα*c)⇔ αCl (λ) ≤ Int (μ) whenever λ ≤ μ and μ is a fuzzy α-open set

(iii) fuzzy generalized α^{**}-closed (in short fgα^{**}c)$\Leftrightarrow$ αCl (λ) $\leq$ Int Cl(μ) whenever $\lambda \leq \mu$ and μ is a fuzzy α-open set.

Definition 2.2[7] Let λ be a fuzzy set in a fuzzy bi topological space(in short fbts) X . The fuzzy set λ is called
(i) a (τ_i, τ_j) fuzzy semi-open(briefly (τ_i, τ_j)-fso)set of X if there exist a $\nu \in \tau_i$ such that $\nu \leq \lambda \leq \tau_j$-Cl($\nu$)
(ii) a (τ_i, τ_j) fuzzy semi-open(briefly (τ_i, τ_j)-fsc)set of X if there exist a $\nu^c \in \tau_i$ such that τ_j-Int(ν) $\leq \lambda \leq \nu$
The set of all (τ_i, τ_j)-fso(resp. (τ_i, τ_j)-fsc) sets of a fbts X will be denoted by (τ_i, τ_j)-FSO(X)(resp. (τ_i, τ_j)-FSC(X))

Definition 2.3 [8] A mapping f: $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_k, \sigma_l)$ is called (τ_i, τ_j) fuzzy pair wise pre semi closed if the image of every (τ_i, τ_j) fuzzy semi-closed set in X is (τ_i, τ_j) fuzzy semi-closed in Y.

2. (τ_i, τ_j) FUZZY GENERALIZED ρ-CLOSED SETS, Where $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$

Definition 3.1: Let (X, τ_i, τ_j) be a fuzzy bi topological space . A fuzzy set λ in X is called
(i) (τ_i, τ_j) fuzzy generalized α-closed if and only if τ_j -αCl(λ) $\leq \mu$ when ever $\lambda \leq \mu$ and μ is τ_i -fuzzy α-open.
(ii) (τ_i, τ^*_j) fuzzy generalized α-closed if and only if τ_j -Cl(λ) $\leq \mu$ whenever $\lambda \leq \mu$ and μ is τ_i -fuzzy α-open.
(iii) (τ^*_i, τ_j) fuzzy generalized α-closed if and only if τ_j -αCl(λ) $\leq \mu$ whenever $\lambda \leq \mu$ and μ is τ_i -fuzzy open.
(iv) (τ_i, τ_j) fuzzy generalized α^*-closed if and only if τ_j -αCl(λ) $\leq \tau_i$ - Int (μ) when ever $\lambda \leq \mu$and μ is τ_i -fuzzy α-open.
(v) (τ_i, τ_j) fuzzy generalized α^{**}-closed if and only if τ_j -αCl(λ) $\leq \tau_i$ -Int (τ_i-Cl (μ))
whenever $\lambda \leq \mu$ and μ is τ_i -fuzzy α-open

.
Remark 3.2 : By setting $\tau_i = \tau_j$ in the definition 3.1
(a)(τ_i, τ_j) fuzzy generalized α-closed set is fuzzy generalized α-closed set [5]
(b)the definition 3.1 , (ii) and (iii) coincide
(c) (τ_i, τ_j) fuzzy generalized α^*-closed is fuzzy generalized α^* -closed set [5]
(d) (τ_i, τ_j) fuzzy generalized α^{**}-closed is fuzzy generalized α^{**} -closed set [5]

Theorem 3.3: Let (X, τ_i, τ_j) be a fuzzy bi topological space
(i) If γ and δ are (τ_i, τ_j) fuzzy generalized α-closed set then $\gamma \cup \delta$ is also(τ_i, τ_j) fuzzy generalized α-closed set.
(ii) If σ is τ_j – fuzzy α- closed subset of X, then σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iii) If σ isτ_j -fuzzy closed subset of X, then σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iv) If σ is τ_j – fuzzy α- closed subset of X, then σ is (τ_i^*, τ_j) fuzzy generalized α-closed set
(v) If σ is τ_j -fuzzy closed subset of X, then σ is (τ_i, τ_j^*) fuzzy generalized α-closed set.

Proof: (i) Suppose μ be a τ_i –fuzzy α-open set greater than $\gamma \cup \delta$ .Since γ and δ are (τ_i, τ_j) fuzzy generalized α-closed sets, τ_j -αCl (γ) $\leq \mu$ and τ_j -αCl (δ) $\leq \mu$. Alsoτ_j -αCl($\gamma \cup \delta$) $\leq \tau_j$ -αCl (γ) $\cup$ τ_j -αCl (δ) $\leq \mu$ .and hence $\gamma \cup \delta$ is (τ_i, τ_j) fuzzy generalized α-closed set.
(ii) Let μ be a τ_i –fuzzy α-open set such that $\sigma \leq \mu$. Since σ is τ_j–fuzzy α- closed , thus τ_j -αCl(σ) = $\sigma \leq \mu$ and hence σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iii) Since every fuzzy closed set is fuzzy α-closed and using (ii) the proof follows.
(iv) Let μ be a τ_i –fuzzy open set such that $\sigma \leq \mu$. Since σ is fuzzy τ_j–fuzzy α- closed τ_j -αCl(σ)= $\sigma \leq \mu$ and hence σ is (τ_i^*, τ_j) fuzzy generalized α-closed set.
(v) Let μ be a τ_i –fuzzy α-open set such that $\sigma \leq \mu$. Since σ is fuzzy τ_j–fuzzy closed τ_j -Cl(σ)= $\sigma \leq \mu$ and hence σ is (τ_i, τ_j^*) fuzzy generalized α-closed set.
The converse of the Theorem 3.3(ii) , (iii) ,(iv) and (v) are need not be true in the following example.

Example 3.4: (i) (τ_1, τ_2) fuzzy generalized α-closed set need not be τ_2-fuzzy α-closed set. Let X ={a,b} , I =[0 1] and the functions f_1 , f_2, f_3,f_4 : $X \to I$ be defined as $f_1(a) = .5$, $f_1(b) =.3$, $f_2(a) =.6$, $f_2(b) =.5$, $f_3(a)= 1$, $f_3(b)= 0$ and $f_4(a)=1$, $f_4(b)=.6$. Consider τ_1={ 0,1, f_1 , f_2 } and τ_2 = { 0,1, f_3,f_4} now (X, τ_1, τ_2) forms fuzzy bitopological space. In this fuzzy bi topological space the fuzzy set λ: $X \to I$ defined as $\lambda(a)= .9$ and $\lambda(b)= 1$ is (τ_1, τ_2) fuzzy generalized α-closed set but not τ_2-fuzzy α-closed set.
(ii) In the fuzzy bi topological space defined (i) the function μ: $X \to I$ defined as $\mu(a) = .5$ and $\mu(b)=.5$ is (τ_1, τ_2) fuzzy generalized α-closed set but not τ_2-fuzzy closed set.
(iii) The fuzzy set λ defined in (i) is (τ_1*, τ_2) fuzzy generalized α-closed set but not τ_2-fuzzy α-closed set.
(iv) The fuzzy set ν: $X\to I$ defined as $\nu(a)= .4$ and $\nu(b)= .7$ in the fuzzy bi topological space (i) is (τ_1* , τ_2) fuzzy generalized α-closed set but not τ_2-fuzzy closed.

Theorem 3.5 : Let (X, τ_i, τ_j) be a fuzzy bi topological space
(i) If γ is (τ_i, τ_j) fuzzy generalized α *-closed set ,then it is (τ_i, τ_j) fuzzy generalized α-closed set.
(ii) If γ is (τ_i, τ_j) fuzzy generalized α -closed set ,then it is (τ_i, τ_j) fuzzy generalized α**-closed set.
Proof: (i) Let $\gamma \leq \mu$, μ be a fuzzy α-open set in τ_i . Since γ is (τ_i, τ_j) fuzzy generalized α *-closed set , we have τ_j -αCl(γ) $\leq \tau_i$- Int (μ) $\leq \mu$. Hence γ is (τ_i, τ_j) fuzzy generalized α-closed set.
(ii) Let $\gamma \leq \mu$, μ be a fuzzy α-open set in τ_i . Since γ is (τ_i, τ_j) fuzzy generalized α -closed set , we have τ_j -αCl(γ) $\leq \mu \leq \tau_i$ - Int (τ_i -Cl (τ_i - Int (μ))) $\leq \tau_i$ - Int (τ_i -Cl (μ)). Hence γ is (τ_i, τ_j) fuzzy generalized α**-closed set.
Converse of the Theorem 3.5 (i) and (ii) are not true in the following example.

Example 3.6: (i) The (τ_1, τ_2) fuzzy generalized α-closed set need not (τ_1 , τ_2) fuzzy generalized α *-closed set. The fuzzy set ν defined in Example 3.4 (iv) is (τ_1 , τ_2) fuzzy generalized α-closed set but not (τ_1 , τ_2) fuzzy generalized α*-closed set
(ii) Let X ={a,b} , I =[0 1] and the functions f_1, f_2,f_3 : $X \to I$ be defined as $f_1(a) = .9$, $f_1(b) =.6$, $f_2(a)= 1$, $f_2(b)= 0$ and $f_3(a)=1$, $f_3(b)=.6$. Consider τ_1={ 0,1, f_1 } and τ_2 = { 0,1, f_2,f_3} now (X, τ_1, τ_2 } forms fuzzy bitopological space. In this fuzzy bi topological space the fuzzy set λ : $X\to I$ defined as $\lambda(a) =.9$ and $\lambda(b) = 1$ is (τ_1, τ_2) fuzzy generalized α**-closed set but not(τ_1 , τ_2) fuzzy generalized α-closed set.

Theorem 3.7 : Let (X, τ_i, τ_j) be a fuzzy bi topological space
(i) If γ is (τ_i, τ_j) fuzzy generalized α-closed set in (X , τ_i^{α}, τ_j) , then it is. (τ_i, τ_j) fuzzy generalized α-closed set .
(ii) If γ is (τ_i, τ_j) fuzzy generalized α-closed set, then it is (τ_i*, τ_j) fuzzy generalized α-closed set.
(iii) If γ is (τ_i, τ_j*) fuzzy generalized α-closed set , then it is. (τ_i, τ_j) fuzzy generalized α-closed set.
γ

Proof: (i) Let $\gamma \leq \mu$, μ be fuzzy α-open set in τ_i . Since γ is $(\tau_i^{\alpha}, \tau_j)$ fuzzy generalized α -closed set , we have τ_j -αCl(γ) $\leq \mu$. Hence γ is (τ_i, τ_j) fuzzy generalized α-closed set.
(ii) Let $\gamma \leq \mu$, μ be fuzzy -open set in τ_i . Since every fuzzy open set is fuzzy α-open and γ is (τ_i, τ_j) fuzzy generalized α -closed set , we have τ_j -αCl(γ) $\leq \mu$. Hence γ is $(\tau_i *, \tau_j)$ fuzzy generalized α-closed set.
(iii) Let $\gamma \leq \mu$, μ is fuzzy α-open set in τ_i . Since γ is $(\tau_i , \tau_j *)$ fuzzy generalized α-closed set , we have τ_j -αCl(γ) $\leq \tau_j$ -Cl(γ) $\leq \mu$. Hence γ is (τ_i, τ_j) fuzzy generalized α-closed set.
Converse of Theorem 3.7 (ii) and (iii) are not true in the following example.

Example 3.8: (i) Consider the fuzzy bitopological space defined in 3.4 (i). In this fbts the fuzzy set λ: $X \to I$ defined as $\lambda(a) =.9$ and $\lambda(b)=1$ is (τ_1*, τ_2) fuzzy generalized α-closed set but not (τ_1, τ_2) fuzzy generalized α-closed set
(ii) (τ_1, τ_2) fuzzy generalized α-closed set need not be (τ_1, τ_2*) fuzzy generalized α-closed set. Let X ={a,b} , I =[0 1] and the functions f_1 , f_2, f_3,f_4 : $X \to I$ be defined as $f_1(a) = .5$, $f_1(b) =.3$, $f_2(a) =.4$, $f_2(b) =.8$, $f_3(a)= .5$, $f_3(b)= .8$ and $f_4(a)=.4$, $f_4(b)=.3$ .Also f_5 , f_6 : $X \to I$ defined as $f_5(a)=.5$,$f_5(b)= .3$ and $f_6(a)= .6$, $f_6(b)= .5$. Consider τ_1={ 0,1, f_1 , f_2 , f_3,f_4} and τ_2 = { 0, 1, f_5,f_6 } now (X, τ_1, τ_2) forms

fuzzy bi topological space. In this fuzzy bi topological space the fuzzy set $\lambda: X \to I$ defined as $\lambda(a)= 0$ and $\lambda(b)= .3$ is
(τ_1, τ_2) fuzzy generalized α-closed set but not be $(\tau_1, \tau_2{}^*)$ fuzzy generalized α-closed set.

Remark 3.9: Let (X, τ_i, τ_j) be a fuzzy bi topological space . A (τ_i, τ_j) fuzzy generalized α -closed sets are generally not equal to (τ_j, τ_i) fuzzy generalized α - closed sets.

Example 3.10: Consider the fuzzy bi topological space defined in Example 3.8.and the fuzzy set $\lambda: X \to I$ defined as $\lambda(a)= 0$ and $\lambda(b)=.3$ .In this example the (τ_1, τ_2) fuzzy generalized α-closure of λ is λ but (τ_2, τ_1) fuzzy generalized α-closure of λ is(.5, .7) . Hence (τ_1, τ_2) fuzzy generalized α-closed sets are generally not equal to (τ_2, τ_1) fuzzy generalized α-closed sets.

Theorem 3.11: If $\tau_i \subseteq \tau_j$ in (X, τ_i, τ_j) , then $G\rho(\tau_i, \tau_j) \supseteq G\rho(\tau_j, \tau_i)$,where $G\rho(\tau_i, \tau_j)$ denote the class of all (τ_i, τ_j)-fuzzy generalized ρ-closed sets where $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$.
Proof: Let $\rho = \alpha$ and γ be a (τ_j, τ_i) fuzzy generalized α-closed set and μ be a τ_i-fuzzy α-open set such that $\gamma \le \mu$ .Since $\tau_i \subseteq \tau_j$,it follows that μ is τ_j - fuzzy α-open and τ_j -$\alpha Cl(\gamma) \le \tau_i$ -$\alpha Cl(\gamma)$ .Then τ_j -$\alpha Cl(\gamma) \le \mu$. Therefore γ is fuzzy (τ_i, τ_j)- generalized α-closed set .When $\rho \in \{\alpha^*, \alpha^{**}\}$ the proof is similar .

Theorem 3.12: Let (X, τ_i, τ_j) be any fuzzy bi topological space and $\tau_j \subseteq \tau_i$. Let $F\alpha O(\tau_i)$ stand for the family of all τ_i -fuzzy α-open sets of X and $F\alpha C(\tau_j)$ stands for the family of all τ_j- fuzzy α- closed subsets of X. Then the following conditions are equivalent
(i) $F\alpha O(\tau_i) = F\alpha C(\tau_j)$
(ii) The fuzzy set λ of X is (τ_i, τ_j) fuzzy generalized α -closed.

Proof: (i) $\Rightarrow$ (ii) Assume that $F\alpha O(\tau_i) = F\alpha C(\tau_j)$. Consider $\lambda \in I^X$, $\lambda \le \mu$ $\mu \in F\alpha O(\tau_i) \Rightarrow \tau_j$ -$\alpha Cl(\lambda) \le \tau_j$ -$\alpha Cl(\mu) =\mu$. Hence λ is (τ_i, τ_j) fuzzy generalized α -closed.
(ii) $\Rightarrow$ (i) Let us assume that $\lambda \in I^X$ is a (τ_i, τ_j) fuzzy generalized α -closed set in X.
Let $\lambda \in F\alpha O(\tau_i)$ .Since $\lambda \le \lambda$ and λ is (τ_i, τ_j) fuzzy generalized α -closed , τ_j -$\alpha Cl(\lambda) \le \lambda$ but $\lambda \le \tau_j$ -$\alpha Cl(\lambda)$ therefore τ_j -$\alpha Cl(\lambda) =\lambda$ implies $\lambda \in F\alpha C(\tau_j)$
$\Rightarrow F\alpha O(\tau_i) \subseteq F\alpha C(\tau_j)$ -------(1)
Assume that $\lambda \in F\alpha C(\tau_j)$, $1-\lambda \in F\alpha O(\tau_j) \subseteq F\alpha O(\tau_j) \Rightarrow \lambda \in F\alpha O(\tau_i)$ .Hence
$F\alpha C(\tau_j) \subseteq F\alpha O(\tau_i)$-------(2)
From (1) and (2) $F\alpha O(\tau_i) = F\alpha C(\tau_j)$

Theorem 3.13: Let (X, τ_i, τ_j) be any fuzzy bi topological space and λ be any fuzzy set in X. Then the following statements are equivalent.
(i) λ^c is (τ_i, τ_j) fuzzy generalized α -closed
(ii) λ is (τ_i, τ_j) fuzzy generalized α -open

Proof: (i) $\Rightarrow$ (ii) Let $\lambda \ge \mu$ where μ is fuzzy τ_i fuzzy α-closed, $\lambda^c=1-\lambda \le 1-\mu$. Since λ^c is(τ_i, τ_j) fuzzy generalized α-closed, τ_j -$\alpha Cl(\lambda^c) \le 1 - \mu \Rightarrow 1-\tau_j$ -$\alpha Int(\lambda) \le 1-\mu$ .Hence τ_j -$\alpha Int(\lambda) \ge \mu$ when ever $\lambda \ge \mu$, μ is τ_i -fuzzy α- closed. Therefore λ is(τ_i, τ_j) fuzzy generalized α-open.
(ii) $\Rightarrow$ (i) Let $\lambda^c \le \mu$, μ is τ_i -fuzzy α- open $\Rightarrow \lambda \ge 1-\mu$, $1-\mu$ is τ_i -fuzzy α -closed. By (ii)τ_j -α $Int(\lambda) \ge 1-\mu. \Rightarrow 1 - \tau_j$ -$\alpha Int(\lambda) \le \mu$, τ_j -$\alpha Cl(1-\lambda) \le \mu$ where μ is τ_i -fuzzy α -open. Hence λ^c is(τ_i, τ_j) fuzzy generalized α- closed.

Theorem 3.14: Let (X, τ_i, τ_j) be any fuzzy bi topological space .For each point λ of (X, τ_i, τ_j) a singleton set $\{\lambda\}$ is τ_j- fuzzy closed (or) λ^c is (τ_i, τ_j) fuzzy generalized α -closed.

Proof: Suppose $\{\lambda\}$ is not τ_j- fuzzy closed . Since λ^c is not τ_j- fuzzy open , a τ_j- fuzzy open set greater than λ^c is only 1 .Then τ_j -$\alpha Cl(\lambda) \le \tau_j$ -$Cl(\lambda) \le 1$ and hence λ^c is (τ_i, τ_j) fuzzy generalized α -closed.

Theorem 3.15 : Let $\tau_i{}^\alpha$ be the family of all τ_i -α closed set in (X, τ_i, τ_j) i =1,2.Then the following conditions are equivalent
(i) In (X, τ_i, τ_j) $\tau_i{}^\alpha = \tau_j{}^\alpha$.

(ii) λ is (τ_i, τ_j) fuzzy generalized α-closed and (τ_j, τ_i) fuzzy generalized α-closed for any subset λ of X.

Proof: (i) $\Rightarrow$ (ii) Let λ be any fuzzy set in a fbts X and μ be τ_j - fuzzy α -open set such that $\lambda \leq \mu$. Since $\tau_i^{\alpha} = \tau_j^{\alpha}$, τ_j-αCl(λ) $\leq \mu$. That is τ_i -αCl(λ) $\leq \mu$. Hence λ is (τ_j, τ_i) fuzzy generalized α-closed. Similarly we can prove λ is (τ_i, τ_j) fuzzy generalized α-closed.
(ii) $\Rightarrow$ (i) Suppose that λ is τ_j - fuzzy α- open .Since λ is (τ_j, τ_i) fuzzy generalized α-closed , τ_i-αCl(λ) $\leq \lambda$ therefore λ is τ_i- fuzzy α- closed and hence $\lambda \in \tau_i^{\alpha}$.
Conversely let λ be an element in τ_i^{α}, by Theorem 3.3 (ii) λ^c is (τ_i, τ_j) fuzzy generalized α-closed τ_j -αCl(λ^c) $\leq \lambda^c$. Therefore λ^c is τ_j - fuzzy α- closed and hence λ is τ_j - fuzzy α- open .Hence $\tau_i^{\alpha} = \tau_j^{\alpha}$.

Theorem 3.16: Let λ and μ be any two fuzzy sets of a fbts X. If λ is fuzzy generalized α-closed and $\lambda \leq \mu \leq \tau_j$-$\alpha$Cl(λ) then μ is (τ_i, τ_j) fuzzy generalized α-closed.

Proof: If λ is (τ_i, τ_j) fuzzy generalized α-closed set and $\lambda \leq \mu \leq \tau_j$ -αCl(λ), $\Rightarrow \tau_j$ -αCl(μ)$\leq$ τ_j -αCl(λ). Since λ is (τ_i, τ_j) fuzzy generalized α-closed, τ_j -αCl(λ) $\leq \nu$ where ν is τ_i-fuzzy α- open set .Hence τ_j-αCl(μ) $\leq \nu$ when ever $\mu \leq \nu$. There fore μ is (τ_i, τ_j) fuzzy generalized α-closed.
If λ be fuzzy set of X and μ be fuzzy set of Y then $(\lambda \times \mu)(x, y) = \min(\lambda(x), \mu(y))$ for each $(x, y) \in X \times Y$. By[1] let λ be a fuzzy closed set of a fuzzy topological space X and μ be a fuzzy closed set of a fuzzy topological space Y. Then $\lambda \times \mu$ is a fuzzy closed set of the fuzzy product topological space $X \times Y$. By [5]every fuzzy closed set is fuzzy generalized α-closed set.
Also by[1] Let X and Y be fuzzy topological space such that X is product relative to Y then for a fuzzy set λ of X and a fuzzy set μ of Y.
$Cl(\lambda \times \mu) = Cl(\lambda) \times Cl(\mu)$
$Int(\lambda \times \mu) = Int(\lambda) \times Int(\mu)$

Theorem 3.17 : If (X, τ_i, τ_j) and (Y, η_k, η_l) are fuzzy bi topological spaces such that X is a product related to Y, then $\lambda \times \mu$ of a $(\tau_i, \tau_j *)$ fuzzy generalized α-open set λ of X and $(\eta_k, \eta_l *)$–fuzzy generalized α-open set μ of Y is a $(\sigma m, \sigma_n *)$ fuzzy generalized α-open set of the fuzzy product space $(X \times Y, \sigma_m, \sigma_n *)$, where σ_r is the fuzzy product topology generalized by τ_r and η_r .

Proof : Let $\lambda \times \mu \leq \gamma_1 \times \gamma_2$ where γ_1 is τ_i -fuzzy α -open and γ_2 is η_k -fuzzy α- open $\Rightarrow \lambda \leq \gamma_1$ and $\mu \leq \gamma_2$. Since λ is $(\tau_i, \tau_j *)$ fuzzy generalized α-closed , τ_j–Cl (λ) $\leq \gamma_1$ and μ is $(\eta_k, \eta_l *)$ fuzzy generalized α-closed, η_l-Cl (μ) $\leq \gamma_2$ .Now σ_n- Cl($\lambda \times \mu$) $\leq \tau_j$–Cl (λ) $\times \eta_l$- Cl (μ) $\leq \gamma_1 \times \gamma_2$ .Hence $\lambda \times \mu$ is $(\sigma_m, \sigma_n *)$ –fuzzy generalized α- closed.

Definition 3.18: Let λ be a fuzzy set in a fbts X .
(i) The (τ_i, τ_j) fuzzy generalized α- interior of λ is denoted by (τ_i, τ_j) -fgαInt(λ) and is defined by (τ_i, τ_j) -fgαInt(λ) = Sup { δ: $\delta \leq \lambda$, δ is (τ_i, τ_j) -fgα-open} and
(ii) The (τ_i, τ_j) fuzzy generalized α- closure of λ is denoted by (τ_i, τ_j) -fgαCl(λ) and is defined by (τ_i, τ_j) -fgαCl(λ) = Inf { δ: $\delta \geq \lambda$, δ is (τ_i, τ_j) -fgα-closed}

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