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OBLONG NUMBERS AND PYTHAGOREAN TRIANGLES

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ABSTRACT

Oblong Numbers as figuret numbers, which were first studied by the Pythagoreans, are studied in terms of Special Pythagorean Triangles. The perimeters of such Triangles are obtained as oblong numbers. Existence of Pythagorean triangles with two consecutive sides and their perimeters as oblong numbers is also investigated. Few interesting observations are made.

Key words: Pythagorean Triangles; Diophantine equation; Oblong numbers.

Subject Classification Code: 11D09, 11-04 and 11Y50.

1. Introduction

Mathematicians all over the ages have been fascinated by the Pythagorean Theorem and are solving many problems related to it thereby developing Mathematics. Integral solutions of ternary quadratic equation are given by Gopalan, Somnath & Vanitha [1] and Gopalan & Kalinga Rani [2]. Special Pythagorean Triangles are generated by Gopalan & Vijyalakshmi [3] and Gopalan and Devibala [4]. In the Pythagorean Mathematics, Gopalan and Janaki [5] have given perimeter of Pythagorean Triangles as a pentagonal Number. These gave the motivation to explore the existence of Pythagorean triangles with their perimeters as oblong numbers. Such Triangles with two consecutive sides and perimeters as oblong numbers are also studied.

2. Method of Analysis:

The primitive solutions of the Pythagorean Equation,

$$\mathbf{X^2 + Y^2 = Z^2}, \quad \mathbf{(1)}$$

is given by [6]

$$X = m^2 - n^2,\ Y = 2mn,\ Z = m^2 + n^2 \quad \mathbf{(2)}$$

for some integers $\boldsymbol{m, n}$ of opposite parity such that $\boldsymbol{m > n > 0}$ and $(m, n) = 1$.

2.1 Perimeter is an oblong number:

Definition 2.1: A natural number p is called an oblong number if it can be written in the form $\beta(\beta + 1),\ \beta \in N$.

If the perimeter of the Pythagorean Triangle (X, Y, Z) is an oblong number p, then

$$X + Y + Z = \beta(\beta + 1) = p. \quad \mathbf{(3)}$$

By virtue of equation (2), equation (3) becomes

$2m^2 + 2mn = \beta(\beta + 1),\ \beta \in N$.

Or, $2m(m + n) = \beta(\beta + 1)$. **(4)**

2.2 Hypotenuse and one leg are consecutive:

In such cases, $m = n + 1$. **(5)**

This gives equation (4) as

$n + 1)(2n + 2) = \beta(\beta + 1)$. $\beta = 2n + 1$.
(6)

Equations (2), (5) & (6) give solutions of equations (1) in correspondence with equations (3) and (4) i.e.,

$X = 2n + 1$;
$Y = 2n(n + 1)$;
$Z = 2n(n + 1) + 1$. **(7)**

First ten such special Pythagorean Triangles (X, Y, Z) are given in the Table 1 below:

Table 1: Special Pythagorean Triangles

S.N.	**n**	**β**	***p***	**X**	**Y**	**Z**
1	1	3	12	3	4	5
2	2	5	30	5	12	13
3	3	7	56	7	24	25
4	4	9	90	9	40	41
5	5	11	132	11	60	61
6	6	13	182	13	84	85
7	7	15	240	15	112	113
8	8	17	306	17	144	145
9	9	19	380	19	180	181
10	10	21	462	21	220	221

Table 2: Verification of $X^2 + Y^2 = Z^2$ and $X + Y + Z = \beta(\beta + 1)$

S.N.	X^2	Y^2	$X^2 + Y^2$	Z^2	$X + Y + Z = \beta(\beta + 1)$
1	9	16	25	25	12 = 3.4
2	25	144	169	169	30 = 5.6
3	49	576	625	625	56 = 7.8
4	81	1600	1681	1681	90 = 9.10
5	121	3600	3721	3721	132 = 11.12
6	169	7056	7225	7225	182 = 13.14
7	225	12544	12769	12769	240 = 15.16
8	289	20736	21025	21025	306 = 17.18
9	361	32400	32761	32761	380 = 19.20
10	441	48400	48841	48841	462 = 20.21

3. 3D Plots:

Solving the Diophantine equation $2m^2 + 2mn = \beta(\beta + 1)$ i.e., $X + Y + Z = \beta(\beta + 1)$ with the help of ***Mathematica*** for $n < m$; $0 < m < 1000$; $0 < n < 1000$; $0 < \beta < 100000$, we get 2116 solutions. Taking these Pythagorean Triangles (X, Y, Z) as points we get the following graph (Figure 1). For computing the values (X, Y, Z) for Pythagorean Triangles, with perimeters as oblong numbers and two sides consecutive, we solve the Diophantine equation,

. □n + 1) (2n + 2) = β (β + 1). When 0 < n < 1000; 0 < β < 100000, we get only 999 solutions. Again taking these Pythagorean Triangles (X, Y, Z) as points we get the following graph (Figure 2):

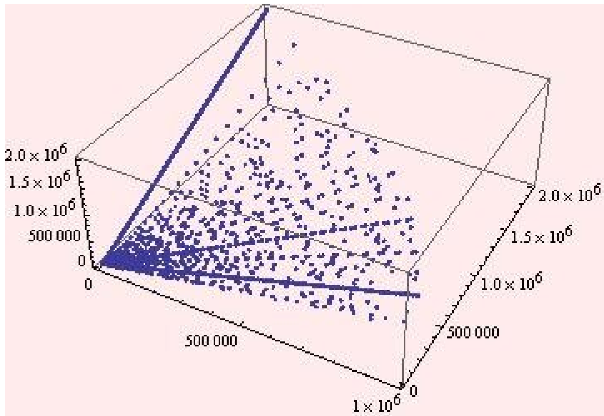

Figure 1

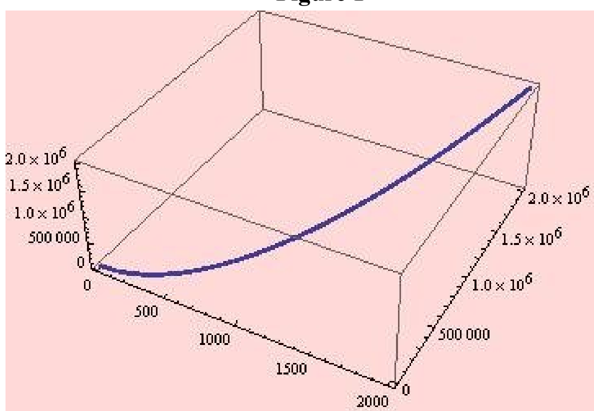

Figure 2

4. Observations and conclusion:

We observe that

1. For each nε N, there corresponds a Pythagorean Triangle with two consecutive sides and perimeter as an oblong number.
2. X + Y + Z = 0(mod 2).
3. The Pythagorean Triplets (X, Y, Z) given by (7) are all primitives as Z = Y + 1.
4. $(Y + Z - X)^2 = 2(Y + Z)(Z - X)$
5. $(X + 2Y + Z)^2 = (Z - X)^2 + 4(X + Y)(Y + Z)$.
6. For each n, values for X and β are the same.

In conclusion, many other patterns of Pythagorean Triangle which satisfy the conditions discussed in the current problem can be found.

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