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Semi-Open Property

Abedallah D. Al-Momany
Ajloun college-Albalqa"a University
Email:abm11168@yahoo.com

Abstract

Let (X,τ) be an extremally disconnected space and $SO(X,\tau)$ be the family of all semi-open subsets of *X*. Then $SO(X,\tau)$ is a topology. A topological property *P* is semi-open provided that (X,τ) has property *P* if and only if $SO(X,\tau) = (X,\tau_{SO})$ has property *P*. In this work we study semi-open property of nearly compact (lindelof), almost compact (lindelöf), weakly lindelöf, weakly compact, s-closed, weakly regular lindelöf, nearly regular lindelöf and almost regular lindelöf. We prove that all these topological properties are semi-open properties.

Keywords: semi-open property, nearly compact, almost compact, weakly compact, nearly lindelöf, almost lindelöf, weakly lindelöf, S-closed, weakly regular lindelöf, almost regular lindelöf and nearly regular lindelöf.

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1. Introduction

In 1969 Signal and Matur [8] introduce and studies the notion of nearly compact spaces. In 1981 Cammaroto and Lo Faro [3] studied weakly compact spaces that defined on covers called regular covers. In 1959 Frolik [5] introduced the notion weakly lindelof spaces. In 1982 Balasubramanian [2] introduce and studied the notion of nearly lindelöf spaces as a generalization of nearly compact spaces. In 1984 Willard and Dissunayake [9] gave the notion of almost lindelöf spaces and in 1996 Cammaroto and Santoro [4] introduce the notion of nearly regular lindelöf,almost regular lindelöf and weakly regular lindelöf by using the regular covers.

The purpose of this paper is to introduce semi-open property and prove that all these topological properties have this property.

Throughout this paper, the interior and the closure of a subset *A* of the topological space (X,τ) will be denoted by *Int(A)* , *cl(A)* respectively. A subset $A \subseteq X$ is called regularly open(regularly closed) if *A=Int(cl(A)) (A=cl(Int(A)))* .A subset $A \subseteq X$ is called semi-open if there exist open set *U* such that $U \subseteq A \subseteq cl(A)$ or equivalently $A \subseteq cl(Int(A))$ and $A \subseteq X$ is called semi-closed if $X \setminus A$ is semi-open or equivalently $Int(cl(A)) \subseteq A$ .The intersection of all semi-closed sets containing *A* is called the semi closure of *A* and is denoted by *scl(A)* .The semi interior of *A* ,denoted by *sint(A)*,is the union of all semi-open sets contained in *A*. The set of all semi-open subsets of a space *X* will be denoted by $SO(X,\tau)$ and the set of all semi closed subsets will be denoted by $SC(X,\tau)$. $A \in SO(X,\tau)$ if *A=sint(A)* and $A \in SC(X,\tau)$ if *A=scl(A)*.For a space (X,τ) we denote by (X,τ_{SO}) the topology on *X* which has $SO(X,\tau)$ as a subbase. A space (X,τ) is called extremally disconnected ,denoted by e.d. , if the closure of any open set is open and it is known that the family $SO(X,\tau)$ is a topology if and only if (X,τ) is extremally disconnected [6] and in this case $SO(X,\tau) = (X,\tau_{SO})$.

Definition 1.1: Let (X,τ) be a topological space, then a topological property *P* is called semi-open property provided that (X,τ) has property *P* if and only if (X,τ_{SO}) has property *P*.

Theorem 1.2 [1]: (1) *sint(A)=A∩cl(Int(A)).*
(2) *scl(A)=A∪Int(cl(A)).*

Lemma 1.3 [7]: If (X,τ) is e.d. then *scl(A)=cl(A)* for any $A \in SO(X,\tau)$.

2. Semi-open property on compact and lindelof spaces

Definition 2.1: A topological space (X,τ) is called:

(1) Compact (lindelöf) if every open cover has a finite (countable) subcover.

(2) Nearly compact (lindelöf) if every open cover $\{U_\alpha | \alpha \in \Delta\}$ of X has a finite (countable) subfamily $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ ($\{U_{\alpha_1}, U_{\alpha_2}, ...\}$) such that $X = \bigcup_{i=1}^{n} Int(cl(U_{\alpha_i}))$ $(X = \bigcup_{i=1}^{\infty} Int(cl(U_{\alpha_i})))$.

(3) Almost compact (lindelöf) if every open cover $\{U_\alpha | \alpha \in \Delta\}$ of X has a finite (countable) subfamily $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ ($\{U_{\alpha_1}, U_{\alpha_2}, ...\}$) such that $X = \bigcup_{i=1}^{n} cl(U_{\alpha_i})$ $(X = \bigcup_{i=1}^{\infty} cl(U_{\alpha_i}))$.

(4) Weakly lindelöf if every open cover $\{U_\alpha | \alpha \in \Delta\}$ of X has a countable subfamily $\{U_{\alpha_1}, U_{\alpha_2}, ...\}$ such that $X = cl(\bigcup_{i=1}^{\infty} U_{\alpha_i})$.

Theorem 2.1: Let (X,τ) be a topological space, then if (X,τ_{SO}) is compact (lindelöf) then (X,τ) is compact (lindelöf).

Proof: Clear since $\tau \subseteq \tau_{SO}$.

The next example shows that the converse of theorem 1.1 is not true and then compact (lindelöf) property is not a semi-open property even if (X,τ) is e.d.

Example 2.2: Let $X=R$ with the topology $\tau = \{\phi, X, \{1\}\}$.Then (X,τ) is compact, lindelöf and e.d.On the other hand the open cover $\{\{x,1\} | x \in X\}$ of (X,τ_{SO}) has no finite (countable) subcover.

Theorem 2.3: Let (X,τ) be e.d. space, then (X,τ) is nearly compact if and only if (X,τ_{SO}) is nearly compact.

Proof : Suppose that (X,τ) is nearly compact and let $\{A_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ_{SO}).For each $\alpha \in \Delta$ there exists $U_\alpha \in \tau$ such that $U_\alpha \subseteq A_\alpha \subseteq cl(U_\alpha)$.Since (X,τ) is e.d. then $cl(U_\alpha)$ is open in (X,τ) for each $\alpha \in \Delta$,so $\{cl(U_\alpha) | \alpha \in \Delta\}$ is an open cover of (X,τ),hence it has a finite subfamily $\{cl(U_{\alpha_1}), cl(U_{\alpha_2}), ..., cl(U_{\alpha_n})\}$such that $X = \bigcup_{i=1}^{n} Int(cl(U_{\alpha_i})) = \bigcup_{i=1}^{n} Int(cl(A_{\alpha_i}))$ $= \bigcup_{i=1}^{n} Int(scl(A_{\alpha_i})) \subseteq \bigcup_{i=1}^{n} \sin t(scl(A_{\alpha_i}))$ }. Hence $X = \bigcup_{i=1}^{n} \sin t(scl(A_{\alpha_i}))$.Thus (X,τ_{SO}) is nearly compact.

Conversely, suppose that (X,τ_{SO}) is nearly compact and let $\{U_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ).Since $\tau \subseteq \tau_{SO}$ then $\{U_\alpha | \alpha \in \Delta\}$ is an open cover of (X,τ_{SO}),so it has a finite subfamily $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ such that $X = \bigcup_{i=1}^{n} \sin t(scl(U_{\alpha_i})) = \bigcup_{i=1}^{n} \sin t(cl(U_{\alpha_i}))$ $= \bigcup_{i=1}^{n} Int(cl(U_{\alpha_i}))$ } because $\{cl(U_{\alpha_i})$ is open for each α_i. Therefore (X,τ) is nearly compact.

Theorem 2.4: Let (X,τ) be e.d. space, then (X,τ) is nearly lindelöf if and only if (X,τ_{SO}) is nearly lindelöf.

Proof: Similar to the proof of theorem 2.3.

Corollary 2.5: Nearly compact (lindelöf) property is a semi-open property.

The condition e.d. in theorems 2.3, 2.4 cannot be dropped as we will see in the next example.

Example 2.6: Let $X=R$ with the topology $\tau = \{U \subseteq X | 0 \notin U\} \cup \{\phi, X\}$, then (X,τ) is nearly compact and not e.d. On the other hand the open cover $\{\{x,0\} | x \in X, x \neq 0\}$ of (X,τ_{SO}) has no finite subfamily $\{x_1,0\},...,\{x_n,0\}$ such that $X = \bigcup_{i=1}^{n} \sin t(scl(\{x_i,0\}))$ because $\sin t(scl(\{x_i,0\}) = \{x_i,0\}$, so (X,τ_{SO}) is not nearly compact.

Theorem 2.7: Let (X,τ) be e.d. space, then (X,τ) is almost compact if and only if (X,τ_{SO}) is almost compact.

Proof : Suppose that (X,τ) is almost compact and let $\{A_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ_{SO}).For each $\alpha \in \Delta$ there exists $U_\alpha \in \tau$ such that $U_\alpha \subseteq A_\alpha \subseteq cl(U_\alpha)$.Since (X,τ) is e.d. then $cl(U_\alpha)$ is open in (X,τ) for each $\alpha \in \Delta$,so $\{cl(U_\alpha) | \alpha \in \Delta\}$ is an open cover of (X,τ) ,hence it has a finite subfamily $\{cl(U_{\alpha_1}), cl(U_{\alpha_2}),...,cl(U_{\alpha_n})\}$such that $X = \bigcup_{i=1}^{n} cl(cl(U_{\alpha_i})) = \bigcup_{i=1}^{n} cl(U_{\alpha_i})$ $= \bigcup_{i=1}^{n} cl(A_{\alpha_i}) = \bigcup_{i=1}^{n} scl(A_{\alpha_i})$. Thus (X,τ_{SO}) is almost compact.

Conversely, suppose that (X,τ_{SO}) is almost compact and let $\{U_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ).Since $\tau \subseteq \tau_{SO}$ then $\{U_\alpha | \alpha \in \Delta\}$ is an open cover of (X,τ_{SO}),so it has a finite subfamily $\{U_{\alpha_1}, U_{\alpha_2},...,U_{\alpha_n}\}$ such that $X = \bigcup_{i=1}^{n} scl(U_{\alpha_i}) = \bigcup_{i=1}^{n} cl(U_{\alpha_i})$. Therefore (X,τ) is almost compact.

Theorem 2.8: Let (X,τ) be e.d. space, then (X,τ) is almost lindelöf if and only if (X,τ_{SO}) is almost lindelöf.

Proof: Similar to the proof of theorem 2.7.

Corollary 2.9: Almost compact (lindelöf) property is a semi-open property.

The condition e.d. in theorems 2.7, 2.8 cannot be dropped as one can see in example 2.6.

Note that if (X,τ) is e.d. then (X,τ) is nearly compact (lindelöf) if and only if (X,τ) is almost compact (lindelöf) and so we have

Corollary 2.10: Let (X,τ) be e.d. space, then the following are equivalent:

(1) (X,τ) is nearly compact (lindelöf).

(2) (X,τ_{SO}) is nearly compact (lindelöf).

(3) (X,τ) is almost compact (lindelöf).

(4) (X,τ_{SO}) is almost compact (lindelöf).

Theorem 2.11: Let (X,τ) be e.d. space, then (X,τ) is weakly lindelöf if and only if (X,τ_{SO}) is weakly lindelöf.

Proof: Suppose that (X,τ) is weakly lindelöf and let $\{A_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ_{SO}).For each $\alpha \in \Delta$ there exists $U_\alpha \in \tau$ such that $U_\alpha \subseteq A_\alpha \subseteq cl(U_\alpha)$. Since (X,τ) is e.d. then $cl(U_\alpha)$ is open in (X,τ) for each $\alpha \in \Delta$, so $\{cl(U_\alpha) | \alpha \in \Delta\}$ is an open cover of (X,τ), hence it has a finite subfamily $\{cl(U_{\alpha_1}), cl(U_{\alpha_2}),...,cl(U_{\alpha_n})\}$such that $X = cl(\bigcup_{i=1}^{\infty} cl(U_{\alpha_i})) = cl(\bigcup_{i=1}^{\infty} U_{\alpha_i})$ $\subseteq cl(\bigcup_{i=1}^{\infty} A_{\alpha_i}) = scl(\bigcup_{i=1}^{\infty} A_{\alpha_i})$ because $\bigcup_{i=1}^{\infty} A_{\alpha_i} \in SO(X,\tau)$,so $X = scl(\bigcup_{i=1}^{\infty} A_{\alpha_i})$.Thus (X,τ_{SO}) is weakly lindelöf.

Conversely, suppose that (X,τ_{SO}) is weakly lindelöf and let $\{U_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ).Since $\tau \subseteq \tau_{SO}$ then $\{U_\alpha | \alpha \in \Delta\}$ is an open cover of (X,τ_{SO}),so it has a countable subfamily $U_{\alpha_1}, U_{\alpha_2},...$such that $X = scl(\bigcup_{i=1}^{\infty} U_{\alpha_i}) = cl(\bigcup_{i=1}^{\infty} U_{\alpha_i})$. Therefore (X,τ) is weakly lindelöf.

Corollary 2.12: Weakly lindelöf property is a semi-open property.

Also the condition e.d. cannot be dropped as one can see in the topological space in example 2.6.

3. Semi-open property on properties with regular cover.

Proposition 3.1: If (X,τ) is an e.d. space, then a subset *C* is regularly closed in (X,τ) if and only if C is regularly closed in (X,τ_{SO}).

Proof: Suppose that *C* is regularly closed in (X,τ), then $C = cl(Int(C))$,hence $C \in SO(X,\tau)$,so $C = \sin t(C)$,and so $C = cl(C) = cl(\sin t(C)) = scl(\sin t(C))$,hence *C* is regularly closed in (X,τ_{SO}).

Conversely, suppose that *C* is regularly closed in (X,τ_{SO}), then $C = scl(\sin t(C)) =$ $cl(\sin t(C)) = cl(C \cap cl(Int(C))) \subseteq cl(C) \cap cl(Int(C)) = cl(Int(C))$, so $C \subseteq cl(Int(C))$. Since *C* is closed in (X,τ) then $cl(Int(C)) \subseteq cl(C) = C$, so $C = cl(Int(C))$, hence C is regularly closed in (X,τ).

Recall that an open cover $\{U_\alpha | \alpha \in \Delta\}$ of a space (X,τ) is called regular [3] if for every $\alpha \in \Delta$ there exists a nonempty regularly closed subset C_α of X such that $C_\alpha \subseteq U_\alpha$ and $X = \bigcup\{Int(C_\alpha) | \alpha \in \Delta\}$.

Definition 3.2 [4]: Let (X,τ) be a topological space, then (X,τ) is called

(1) S-closed if every cover of *X* by regularly closed sets has a finite subcover.

(2) Weakly compact if every regular cover $\{U_\alpha | \alpha \in \Delta\}$ of (X,τ) has a finite subfamily $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ such that $X = \bigcup_{i=1}^{n} cl(U_{\alpha_i})$.

(3) Almost regular lindelöf (Weakly regular lindelöf, Nearly regular lindelöf) if every regular cover $\{U_\alpha | \alpha \in \Delta\}$ has a countable subfamily $U_{\alpha_1}, U_{\alpha_2}, ...$ such that $X = \bigcup_{i=1}^{\infty} cl(U_{\alpha_i})$ ($X = cl(\bigcup_{i=1}^{\infty} U_{\alpha_i})$, $X = \bigcup_{i=1}^{\infty} Int(cl(U_{\alpha_i}))$).

Theorem 3.3: Let (X,τ) be an e.d. space, then (X,τ) is S-closed if and only if (X,τ_{SO}) is S-closed.

Proof: Clear using proposition 3.1.

Corollary 3.4: s-closed property is a semi-open property.

Theorem 3.5: Let (X,τ) be e.d. space, then (X,τ) is weakly compact if and only if (X,τ_{SO}) is weakly compact.

Proof: Suppose that (X,τ) is weakly compact and let $\{A_\alpha | \alpha \in \Delta\}$ be a regular cover of (X,τ_{SO}).For each $\alpha \in \Delta$ there exists $U_\alpha \in \tau$ such that $U_\alpha \subseteq A_\alpha \subseteq cl(U_\alpha)$ and there exists a regularly closed set C_α in (X,τ_{SO}) such that $C_\alpha \subseteq A_\alpha \subseteq cl(U_\alpha)$ with $X = \bigcup\{\sin t(C_\alpha) | \alpha \in \Delta\}$ $= \bigcup\{C_\alpha \cap cl(Int(C_\alpha)) | \alpha \in \Delta\}$.By proposition 3.1 , C_α is regularly closed in (X,τ), so $X = \bigcup\{cl(Int(C_\alpha)) | \alpha \in \Delta\} = \bigcup\{Int(cl(Int(C_\alpha))) | \alpha \in \Delta\} = \bigcup\{Int(C_\alpha) | \alpha \in \Delta\}$, hence $\{cl(U_\alpha) | \alpha \in \Delta\}$ is a regular cover of (X,τ) ,so it has a finite subfamily $\{cl(U_{\alpha_1}), cl(U_{\alpha_2}), ..., cl(U_{\alpha_n})\}$ such that $X = \bigcup_{i=1}^{n} cl(cl(U_{\alpha_i})) = \bigcup_{i=1}^{n} cl(U_{\alpha_i}) = \bigcup_{i=1}^{n} cl(A_{\alpha_i}) = \bigcup_{i=1}^{n} scl(A_{\alpha_i})$.

Therefore (X,τ_{SO}) is weakly compact.

Conversely, suppose that (X,τ_{SO}) is weakly compact and let $\{U_\alpha | \alpha \in \Delta\}$ be an open cover of (X,τ). For each $\alpha \in \Delta$ there exists a regularly closed set C_α in (X,τ) such that $C_\alpha \subseteq U_\alpha$ with $X = \bigcup\{Int(C_\alpha) | \alpha \in \Delta\} \subseteq \bigcup\{\sin t(C_\alpha) | \alpha \in \Delta\}$. By proposition 3.1 C_α is regularly closed in (X,τ_{SO}) for each $\alpha \in \Delta$, so $\{U_\alpha | \alpha \in \Delta\}$ is a regular cover of (X,τ_{SO}), so it has a finite subfamily $\{U_{\alpha_1}, U_{\alpha_2}, ..., U_{\alpha_n}\}$ such that $X = \bigcup_{i=1}^{n} scl(U_{\alpha_i}) = \bigcup_{i=1}^{n} cl(U_{\alpha_i})$. Therefore (X,τ) is weakly compact.

Theorem 3.6: Let (X,τ) be e.d. space, then (X,τ) is almost regular lindelöf if and only if (X,τ_{SO}) is almost regular lindelöf.

Proof: Similar to the proof of theorem 3.5.

Corollary 3.7: Weakly compact and almost regular lindelöf properties are semi-open properties.

The condition e.d. in theorems 3.5 and 3.6 cannot be dropped as we can see in the next example.

Example 3.8: The topological space in example 2.6 is weakly compact and the cover $\{\{x,0\} | x \in X, x \neq 0\}$ is a regular cover of (X,τ_{SO}) because $\{x,0\}$ is regularly closed in (X,τ_{SO}) and $\sin t(\{x,0\}) = \{x,0\}$. But this cover has no countable subfamily $\{x_1,0\}, \{x_2,0\}, ...$ such that $X = \bigcup_{i=1}^{\infty} scl(\{x_i,0\}))$.

Theorem 3.9: Let (X,τ) be e.d. space, then (X,τ) is weakly regular lindelöf if and only if (X,τ_{SO}) is weakly regular lindelöf.

Proof: Suppose that (X,τ) is weakly regular lindelöf and let $\{A_\alpha | \alpha\in\Delta\}$ be a regular cover of (X,τ_{SO}). For each $\alpha\in\Delta$ there exists $U_\alpha\in\tau$ such that $U_\alpha\subseteq A_\alpha\subseteq cl(U_\alpha)$. As in the proof of theorem 3.5, $\{cl(U_\alpha) | \alpha\in\Delta\}$ is a regular cover of (X,τ), so it has a countable subfamily $\{cl(U_{\alpha_1}), cl(U_{\alpha_2}),\ldots\}$ such that $X = cl(\bigcup_{i=1}^{\infty} cl(U_{\alpha_i})) = cl(\bigcup_{i=1}^{\infty} U_{\alpha_i}) \subseteq cl(\bigcup_{i=1}^{\infty} A_{\alpha_i}) = scl(\bigcup_{i=1}^{\infty} A_{\alpha_i})$. Thus (X,τ_{SO}) is weakly regular lindelöf.

Conversely, suppose that (X,τ_{SO}) is weakly regular lindelöf and let $\{U_\alpha | \alpha\in\Delta\}$ be a regular cover of (X,τ). As in the proof of theorem 3.5, $\{U_\alpha | \alpha\in\Delta\}$ is a regular cover of (X,τ_{SO}), so it has a countable subfamily $\{U_{\alpha_1}, U_{\alpha_2},\ldots\}$ such that $X = scl(\bigcup_{i=1}^{\infty} U_{\alpha_i}) = cl(\bigcup_{i=1}^{\infty} U_{\alpha_i})$. Therefore (X,τ) is weakly regular lindelöf.

Corollary 3.10: weakly regular lindelöf is a semi-open property.

Also the condition e.d. in theorems 3.9 cannot be dropped as one can see in example 3.8.

. *Theorem* 3.11: Let (X,τ) be e.d. space, then (X,τ) is nearly regular lindelöf if and only if (X,τ_{SO}) is weakly regular lindelöf.

Proof: Suppose that (X,τ) is nearly regular lindelöf and let $\{A_\alpha | \alpha\in\Delta\}$ be a regular cover of (X,τ_{SO}). For each $\alpha\in\Delta$ there exists $U_\alpha\in\tau$ such that $U_\alpha\subseteq A_\alpha\subseteq cl(U_\alpha)$. As in the proof of theorem 3.5, $\{cl(U_\alpha) | \alpha\in\Delta\}$ is a regular cover of (X,τ), so it has a countable subfamily $\{cl(U_{\alpha_1}), cl(U_{\alpha_2}),\ldots\}$ such that

$X = \bigcup_{i=1}^{\infty} Int(cl(cl(U_{\alpha_i}))) = \bigcup_{i=1}^{\infty} Int(cl(U_{\alpha_i})) = \bigcup_{i=1}^{\infty} \sin t(scl(U_{\alpha_i})) \subseteq \bigcup_{i=1}^{\infty} \sin t(scl(A_{\alpha_i}))$. Thus (X,τ_{SO}) is nearly regular lindelöf.

Conversely , suppose that (X,τ_{SO}) is nearly regular lindelöf and let $\{U_\alpha | \alpha\in\Delta\}$ be a regular cover of (X,τ). As in the proof of theorem 3.5 , $\{U_\alpha | \alpha\in\Delta\}$ is a regular cover of (X,τ_{SO}), so it has a countable subfamily $\{U_{\alpha_1}, U_{\alpha_2},\ldots\}$ such that $X = \bigcup_{i=1}^{\infty} \sin t(scl(U_{\alpha_i})) = \bigcup_{i=1}^{\infty} \sin t(cl(U_{\alpha_i})) = \bigcup_{i=1}^{\infty} Int(cl(U_{\alpha_i}))$ because $\sin t(cl(U_{\alpha_i})) = Int(cl(U_{\alpha_i}))$ since $cl(U_{\alpha_i})\in\tau$.

Therefore (X,τ) is nearly regular lindelöf.

Corollary 3.12: Nearly regular lindelöf is a semi-open property.

Also the condition e.d. in theorems 3.11 cannot be dropped as one can see in example 3.8.

References:

(1) D. Andrijeric , *Semi-preopen sets* , Math. Vesnik , 38 (1) (1986) 24-32.

(2) G. Balasubramanian ,*On some generalizations of compact spaces* , Glasnik Math. ,17 (37) (1982) 367-380.

(3) F. Cammaroto and G. Lo Faro , *Spazi weakly compact* , Riv. Math. University Parma ,14 (7) (1981) 383-395.

(4) F. Cammaroto and G. Santoro , *Some counterexamples and properties on generalizations of lindelöf spaces* , Internat. J. Math. Sci. ,19 (4) (1996) 737-746.

(5) Z. Frolik , *Generalization of compact and lindelöf spaces* .Czcchoslovak Math. J. , 9 (84) (1959) 702-710.

(6) O. Njastad , *On some class of nearly open sets* , Pacific J. Math. , 15 (1965) 961-970.

(7) T. Noiri , *Properties of s-closed spaces* , Acta Math. Hungar. ,35 (1980) 431-436.

(8) M. K. Singal and A. Mathur , *On nearly compact spaces* ,Bull. Un. Math. Ital. ,4 (2) (1969) 702-710.

(9) S. Willard and U. N. B. Dissanayake ,*The almost lindelöf degree* , Canad. Math. Bull. , 27 (4) (1984) 452-455.