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TWO NEW WEIGHTED MEASURES OF FUZZY ENTROPY AND THEIR PROPERTIES

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ABSTRACT

Measures of weighted information are well known in the literature of information theory. In the present communication, we have developed some new generalized measures of weighted fuzzy entropy, studied their important properties for their authenticity and have presented them graphically.

Keywords: Fuzzy entropy, Fuzzy set, Symmetry, Concavity, Trigonometric entropy.

INTRODUCTION

The notion of fuzzy set stems from the observation made by Zadeh [16] that “more often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership”. This observation emphasizes the gap existing between mental representations of reality and usual mathematical representations thereof, which are based on binary logic, precise numbers and the like. According to Zadeh [16] classes of objects exist only through such mental representations through natural language terms such as high temperature, young man, big size, etc., and also with nouns such as bird, chair, etc. When proposing fuzzy sets, Zadeh’s [16] concerns were explicitly centered on their potential contribution in the domains of pattern classification, processing and communication of information, abstraction and summarization. Thus, keeping in view the idea of fuzzy sets, De Luca and Termini [2] introduced the following measure of fuzzy entropy corresponding to Shannon’s [15] measure.

$$H(A) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i) + \left(1-\mu_A(x_i)\right)\log\left(1-\mu_A(x_i)\right)\right] \tag{1.1}$$

After this development, a large number of measures of fuzzy entropy were discussed, characterized and generalized by various authors. Kapur [7] introduced the following measure of fuzzy entropy:

$$H_{\alpha,\beta}(A) = \frac{1}{\beta-\alpha}\log\frac{\sum_{i=1}^{n}\left\{\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha}\right\}}{\sum_{i=1}^{n}\left\{\mu_A^{\beta}(x_i) + (1-\mu_A(x_i))^{\beta}\right\}};\alpha \geq 1, \beta \leq 1 \tag{1.2}$$

Parkash [9] introduced a new generalized fuzzy entropy involving two real parameters, given by

$$H_{\alpha}^{\beta}(A) = [(1-\alpha)\beta]^{-1}\sum_{i=1}^{n}\left[\left\{\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha}\right\}^{\beta} - 1\right]; \alpha > 0, \alpha \neq 1, \beta \neq 0 \tag{1.3}$$

and called it ($\alpha-\beta$) fuzzy entropy which includes some well known measures of fuzzy entropy.
Parkash and Tuli [11] introduced the following trigonometric measure of entropy:

$$H_s(A)=\sum_{i=1}^{n}\mathrm{Sin}\left\{\frac{2k(n-2)\pi}{n}\mu_A(x_i)+\alpha\right\}+\sum_{i=1}^{n}\mathrm{Sin}\left\{\frac{2k(n-2)\pi}{n}(1-\mu_A(x_i))+\alpha\right\}-n\mathrm{Sin}\alpha$$

$$-n\mathrm{Sin}\left\{\frac{2k(n-2)\pi}{n}\right\}$$

(1.4)

where k and α are two parameters satisfying

$$0<\alpha<\pi,\ 0<k\le 1-\frac{\alpha}{\pi}$$

(1.5)

Parkash and Tuli [11] have provided the applications of the fuzzy measures of entropy to the field of trigonometry and obtained the following trigonometric inequalities:

$$\sum_{i=1}^{n}\mathrm{Sin}(kA_i+\alpha)\le n\mathrm{Sin}\left\{\frac{k(n-2)\pi}{n}+\alpha\right\} \qquad (1.6)$$

The equality sign in (1.6) holds only when $A_1=A_2=\ldots A_n=\frac{n-2}{n}\pi$

The inequality (1.6) is a basic trigonometric inequality involving trigonometric functions of the angles $A_1, A_2, \ldots, A_n$ of a convex polygon of n sides. The equality sign holds when all angles are equal.

Moreover, Parkash and Tuli [12] obtained the expressions for optimum fuzziness in different measures of fuzzy entropy. Parkash, Sharma and Mahajan [10] developed some new measures of weighted fuzzy entropy and provided their applications for the study of maximum weighted fuzzy entropy principle. Some other measures of fuzzy entropy have been studied by Ebanks [3], Rudas [14], Kapur [6], Guo and Xin [4], Parkash and Tuli [13], Hu and Yu [5], Pal and Bezdek [8], Osman, Abdel-Fadel, El-Sersy, and Ibraheem [7] etc.

The concept of weighted information which was introduced by Belis and Guiasu [1], received a good response from information theoreticians and this introduction opened a new field for researchers who started working over the concept of weighted information. Thus, on attaching weights to the fuzzy values, measure of weighted fuzzy entropy can be constructed parallel to the fuzzy entropy introduced by De Luca and Termini [2]. This measure is given by

$$H(A;W)=-\sum_{i=1}^{n}w_i\left[\mu_A(x_i)\log\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\log\left(1-\mu_A(x_i)\right)\right] \qquad (1.7)$$

Using the concept of this weighted, we have developed two new measures of fuzzy entropy, the findings of which have been presented in the next section.

2. TWO NEW WEGHTED MEASURES OF FUZZY ENTROPY

I. Firstly, we propose a new non-parametric weighted measure of fuzzy entropy as given by the following mathematical expression:

$$H_1(A:W)=\frac{n}{\sum_{i=1}^{n}w_i\left[\frac{1}{\mu_A(x_i)}+\frac{1}{1-\mu_A(x_i)}\right]} \qquad (2.1)$$

The result (2.1) can be written as

$$H_1(A:W)=\frac{n}{\sum_{i=1}^{n}w_i\left[\frac{1}{x_i(1-x_i)}\right]} \qquad (2.2)$$

where $x_i = \mu_A(x_i); i = 1, 2, ..., n$

Differentiating equation (2.2) twice, we get $\frac{\partial^2 H_1(A:W)}{\partial x_i^2} < 0$

So $H_1(A:W)$ is a concave function of x_i.

Thus, we note that $H_1(A:W)$ satisfies the following properties:

(i) $H_1(A:W)$ is a concave function of $\mu_A(x_i)$.

(ii) $H_1(A:W)$ is an increasing function of $\mu_A(x_i)$ when $0 \le \mu_A(x_i) \le \frac{1}{2}$

(iii) $H_1(A:W)$ is a decreasing function of $\mu_A(x_i)$ when $\frac{1}{2} \le \mu_A(x_i) \le 1$

(iv) $H_1(A:W)$ does not change when $\mu_A(x_i)$ is changed to 1- $\mu_A(x_i)$.

(v) $H_1(A:W)$ = 0 when $\mu_A(x_i)$ = 0 or 1

Thus, $H_1(A:W)$ is a valid measure of weighted fuzzy entropy.

Note: To obtain maximum value of $H_1(A:W)$, we have

$$\frac{\partial H_1(A:W)}{\partial \mu_A(x_i)} = 0 \Rightarrow \mu_A(x_i) = \frac{1}{2}$$

$$\text{Thus } \left(H_1(A:W)\right)_{max} = \frac{n}{\sum_{i=1}^{n} w_i \left[\frac{1}{1/2} + \frac{1}{1/2}\right]} = \frac{n}{4\sum_{i=1}^{n} w_i}$$

Next, with the help of numerical data, we have presented the weighted measure (2.1) graphically. For this purpose, we have computed different values of $H_1(A:W)$ corresponding to different fuzzy values $\mu_A(x_i)$ under the weighted distribution $W = \{8, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 9\}$ as shown in the following Table- 2.1:

Table-2.1

$\mu_A(x_i)$	w_i	$H_1(A:W)$
0.0	8.0	0.0000
0.1	8.1	0.0111
0.2	8.2	0.0195
0.3	8.3	0.0253
0.4	8.4	0.0286
0.5	8.5	0.0294
0.6	8.6	0.0279
0.7	8.7	0.0241
0.8	8.8	0.0182
0.9	8.9	0.0101
1.0	9.0	0.0000

Next, we have presented $H_1(A:W)$ graphically and obtained Fig.-2.1 which shows that the weighted measure (2.1) is a concave function and this proves the authenticity of the proposed measure.

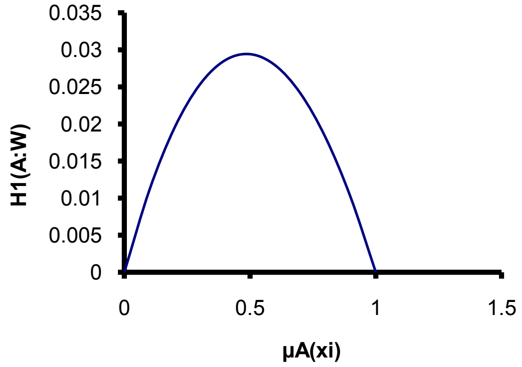

Fig.-2.1

II. Next, we propose another new weighted measure of fuzzy entropy as follow:

$$H_2^{\alpha}(A:W) = -\frac{1}{n}\sum_{i-1}^{n} w_i\left[(n\mu_A(x_i)-1)^{\alpha} + \{n(1-\mu_A(x_i))-1\}^{\alpha} - (-1)^{\alpha} - (n-1)^{\alpha}\right] \tag{2.3}$$

Proceeding as above, we have checked that $H_2^{\alpha}(A:W)$ satisfies the following properties:

(i) $H_2^{\alpha}(A:W)$ is a concave function of $\mu_A(x_i)$.

(ii) $H_2^{\alpha}(A:W)$ doesn't change when $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$.

(iii) $H_2^{\alpha}(A:W)$ is an increasing function of $\mu_A(x_i)$ for $0 \le \mu_A(x_i) \le \frac{1}{2}$.

(iv) $H_2^{\alpha}(A:W)$ is a decreasing function of $\mu_A(x_i)$ for $\frac{1}{2} \le \mu_A(x_i) \le 1$.

(v) $H_2^{\alpha}(A:W) = 0$ when $\mu_A(x_i)$ = 0 or 1.

Hence, $H_2^{\alpha}(A:W)$ is a valid measure of fuzzy entropy.

Next, we have presented the values of $H_2^{\alpha}(A:W)$ graphically and obtained the following Fig.-2.2 which shows that the measure introduced in equation (2.3) is a concave function.

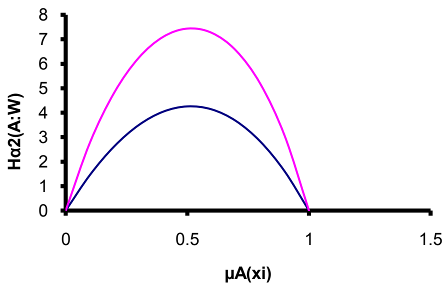

Fig.-2.2

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