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On soft fuzzy locally C-precompactification

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Abstract: Two notions of local C-precompactness in the soft fuzzy C-convergence space are introduced. Moreover for each given soft fuzzy C-convergence space, there exists a coarsest locally C-precompact space which is finer than the original space. Soft fuzzy locally C-precompactification and soft fuzzy C-Tychonoff theorem in a soft fuzzy locally C-precompact convergence space is established.

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1 Introduction:

The concept of fuzzy subset was introduced by Zadeh [13]. Choquet [3] approached the ideas of convergence, continuity, connectedness, e.t.c., using convergence instead of neighbourhood. The concept of soft fuzzy subset was introduced by Ismail U. Tiryaki [9]. Several properties on fuzzy product topological spaces were discussed by K.K. Azad [1]. The soft fuzzy C-open set and soft fuzzy product topological space were introduced by T.Yogalakshmi, E.Roja, M.K.Uma [10, 11]. The concepts of soft fuzzy C-convergence space was introduced and developed by T.Yogalakshmi, E.Roja, M.K.Uma [12].

In this paper, soft fuzzy locally C-precompact convergence space are introduced. compactification and soft fuzzy Tychonoff theorem in a soft fuzzy locally C-precompact convergence space is established.

2 Preliminaries :

Definition: 2.1. [10] Let X be a nonempty set and I=[0,1] be the unit interval. Let μ be a fuzzy subset of X such that $\mu : X \to [0,1]$ and M be any crisp subset of X.Then, the pair (μ, M) is called as a **soft fuzzy set** in X. The family of all soft fuzzy subsets of X,will be denoted by **SF(X)**.

Definition: 2.2. [10] Let $x \in X$ and $\lambda : X \to [0,1]$. Define,

$$x_\lambda(y) = \begin{cases} \lambda\ (0 < \lambda \leq 1), & if x = y \\ 0, & otherwise \end{cases}$$

Then, the soft fuzzy set $(x_\lambda, \{x\})$ is called as the **soft fuzzy point** (inshort, **SFP**)in $SF(X)$, with **support, x** and **value,** λ.

Definition: 2.3. [10] A **soft fuzzy topology** on a non-empty set X is a family τ of soft fuzzy sets in X satisfying the following axioms:

(1) $(0,\phi),(1,X) \in \tau$.

(2) For any family of soft fuzzy sets $(\lambda_j, N_j) \in \tau, j \in J$, $\Rightarrow \sqcup_{j\in J}(\lambda_j, N_j) \in \tau$.

(3) For any finite number of soft fuzzy sets $(\lambda_j, N_j) \in \tau$, j=1,2,3,...n , $\Rightarrow \sqcap_{j=1}^{n}(\lambda_j, N_j) \in \tau$.

Then, the pair $(X,\tau)$is called as a **soft fuzzy topological space**. (in short, **SFTS**). The members of τ is said to be a **soft fuzzy open set** (in short, **SFOS**) in X. The complement of SFOS is said to be a **soft fuzzy closed set**, (inshort, **SFCS**) in X.

Definition: 2.4. [10] Let (X,τ) be a SFTS. A soft fuzzy set (λ, N) is said to be **soft fuzzy C-open** , if

$$(\lambda, N)=(\mu, M) \sqcap (\delta, L)$$

where,(μ, M) is a soft fuzzy open set and (δ, L) is a soft fuzzy α^*-open set.

The complement of soft fuzzy C-open set (in short. **SFcOS**) is called as a **soft fuzzy C-closed set**. (in short.**SFcCS**). The family of all soft fuzzy C-open set is denoted by $SF_C(X,\tau)$.

Definition: 2.5. [11] Let $X = X_1 \times X_2$ be the product space. Let (λ_1, N_1) and (λ_2, N_2) be any soft fuzzy sets in X_1 and X_2 respectively. Then, the **soft fuzzy product set** on X is defined as $(\lambda_1, N_1) \times (\lambda_2, N_2)$ = $(\lambda_1 \times \lambda_2, N_1 \times N_2)$, where $\lambda_1 \times \lambda_2$ is the fuzzy product set and $N_1 \times N_2$ is the cartesian product of N_1 and N_2.

Definition: 2.6. [11] Let $\mathbb{X} = \prod_{j\in J} X_j$ be the product space and $(\mathbb{X},\tau)$ be the soft fuzzy product topological space. Let $(\gamma, K) = \prod_{j\in J}(\gamma_j, K_j)$ be the product set on $\mathbb{X}$ and each $(\gamma_j, K_j) \in X_j$. Let $((\gamma_j, K_j), lim_{(\gamma_j,K_j)}) \in |SFC\text{-}CS|$, $j \in J$. Define the **product structure** $lim_{(\gamma,K)} : \mathbb{F}_{C_X}((\gamma, K)) \to SF_X((\gamma, K))$ as,

$$lim_{(\gamma,K)}\ \mathfrak{F}_c = \begin{cases} \sqcap_{j\in J} lim_{(\gamma_j,K_j)} P_j^{(\gamma,K)} \mathfrak{F}_c, & \text{when } \mathfrak{F}_c \in \mathbb{P}_{C_{\mathbb{X}}}(\gamma, K) \\ \sqcap_{\mathfrak{G}_c \in \mathbb{P}_{(\gamma,K)_m}(\mathfrak{F}_c)} lim_{(\gamma,K)} \mathfrak{G}_c, & \text{otherwise} \end{cases}$$

where, $P_j^{(\gamma,K)} : (\gamma, K) \to (\gamma_j, K_j)$ is the j^{th} projection map and $\mathbb{F}_{C_{\mathbb{X}}}((\gamma, K))$ is the set of all soft fuzzy C-filters on $(\gamma, K) \in SF_C(\mathbb{X},\tau)$.

Definition: 2.7. [12] Let (X,τ) be a SFTS. A ***soft fuzzy C-filter***, $\mathfrak{F}_c$ on $(\gamma, K) \in SF_C(X,\tau)$ is a non-empty collection of soft fuzzy C-open sets in $SF_{C_X}((\gamma, K))$ provided,

(1) $(\lambda, N) \notin \mathfrak{F}_c$ such that either $\lambda = 1_\phi$ or $N = \phi$.

(2) $(\mu, M), (\lambda, N) \in \mathfrak{F}_c \Rightarrow (\mu, M) \sqcap (\lambda, N) \in \mathfrak{F}_c$.

(3) $(\lambda, N) \in \mathfrak{F}_c$ and $(\lambda, N) \sqsubseteq (\mu, M) \sqsubseteq (\gamma, K) \Rightarrow (\mu, M) \in \mathfrak{F}_c$.

The family of all soft fuzzy C-filters on (γ, K) is denoted by $\mathbb{F}_{C_X}((\gamma, K))$.

Definition: 2.8. [12] A soft fuzzy C-filter $\mathfrak{F}_c$ on $(\gamma, K) \in SF_C(X,\tau)$ is said to be a ***soft fuzzy prime C-filter*** , whenever $(\lambda, N) \sqcup (\mu, M) \in \mathfrak{F}_c$ with either $\widehat{(\lambda, N)} \neq \phi$ or $\widehat{(\mu, M)} \neq \phi \Rightarrow (\mu, M) \in \mathfrak{F}_c$ or $(\lambda, N) \in \mathfrak{F}_c$. The family of all soft fuzzy prime C-filters on (γ, K), is denoted by $\mathbb{P}_{C_X}(\gamma, K)$. Also, the family of all soft fuzzy prime C-filters containing $\mathfrak{F}_c$ on (γ, K) is denoted by $\mathbb{P}_{(\gamma,K)}(\mathfrak{F}_c)$.

Notation: $\widehat{(\lambda, N)} = \{x \in X : \lambda(x) > 0 \text{ and } x \in N\}$ and $(\lambda, N)_0 = \{x \in X : \lambda(x) > 0 \text{ or } x \in N\}$.

Definition: 2.9. [12] A ***soft fuzzy principal C-filter*** $(\dot{\mu}, \dot{M})$, is the soft fuzzy C-filter on (γ, K), generated by (μ, M),

$$(\mu, M)^{\cdot}_{(\gamma,K)} := (\dot{\mu}, \dot{M}) := \{(\lambda, N) \in SF_{C_X}(\gamma, K) : (\mu, M) \sqsubseteq (\lambda, N) \sqsubseteq (\gamma, K)\}.$$

Definition: 2.10. [12] Let $\mathfrak{F}_c$ be a soft fuzzy C-filter on (γ, K) and $(\beta, L) \sqsubseteq (\gamma, K)$. The **induced soft fuzzy C-filter on** (β, L) is defined by

$$(\mathfrak{F}_c)|_{(\beta,L)} := \{(\lambda,N)\sqcap(\beta,L) : (\lambda,N)\in\mathfrak{F}_c \text{ with } \lambda\wedge\beta\neq 1_\phi \text{ or } N\cap L\neq\phi\}$$

Definition: 2.11. [12] The **characteristic set** of a soft fuzzy C-filter $\mathfrak{F}_c$ on (γ,K) is defined by

$$\mathfrak{c}(\mathfrak{F}_c) = \left(\wedge_{(\mu,M)\in\mathfrak{F}_c}\vee_{x\in X}\mu(x), \cap_{(\mu,M)\in\mathfrak{F}_c}M\right)$$

Definition: 2.12. [12] A soft fuzzy topological space (X,τ) is said to be a **soft fuzzy $F^\sharp$ space**, if for each $x\in X$, there exists all soft fuzzy points $(\alpha 1_x,\{x\})\in\tau$, where $0<\alpha\leq 1$.

Definition: 2.13. [12] The **multifunction** is a function which gives the connection between soft fuzzy C-filter and filter, established by the mappings

$$\iota : \begin{cases} \mathbb{F}_X((\gamma,K))\to F((\gamma,K)_0) \\ \mathfrak{F}_c\to\iota\mathfrak{F}_c := \{(\mu,M)_0 : (\mu,M)\in\mathfrak{F}_c\}\end{cases}$$

And,

$$\omega : \begin{cases} F((\gamma,K)_0)\to\mathbb{F}_X((\gamma,K)) \\ \mathcal{F}\to\omega\mathcal{F} := (\{(1_A,A)\sqcap(\gamma,K) : A\in\mathcal{F}\})^{\cdot}_{(\gamma,K)}\end{cases}$$

Proposition: 2.1. [12] Let (X,τ) be a soft fuzzy $F^\sharp$ spaces. Let $\mathfrak{F}_c$ be a soft fuzzy C-filter on (γ,K) and $\mathcal{F}$ on $(\gamma,K)_0$. Let $f : X\to Y$ be a function. Then, the following relations are valid.

1. ι and ω are respectively an isotone surjection and injection.
2. $\iota\omega\mathcal{F}=\mathcal{F}, \mathcal{F}\in F_X((\gamma,K)_0)$.
3. $\omega\iota\mathfrak{F}_c\sqsubseteq\mathfrak{F}_c, \mathfrak{F}_c\in\mathbb{F}_X((\gamma,K))$.
4. $\mathfrak{F}_c\in\mathbb{F}_X((\gamma,K))$ is a soft fuzzy prime C-filter on (γ,K) iff $\iota\mathfrak{F}_c$ is an ultrafilter on $(\gamma,K)_0$.
5. $\mathcal{F}\in F((\gamma,K)_0)$ is an ultrafilter iff $\omega\mathcal{F}$ is a soft fuzzy prime C-filter on (γ,K).
6. $\mathbb{P}_{(\gamma,K)_m}(\mathfrak{F}_c) = \{\omega\mathcal{U}\sqcup\mathfrak{F}_c : \mathcal{U}\supseteq\iota\mathfrak{F}_c$, for some ultrafilter $\mathcal{U}$ on $(\gamma,K)_0\}$.
7. If for each $\mathfrak{G}_c\in\mathbb{P}_{(\gamma,K)_m}(\mathfrak{F}_c)$, then for some finite subcollection $\mathfrak{G}_{c_1},\mathfrak{G}_{c_2},...\mathfrak{G}_{c_n}$,

$$\sqcup_{\substack{1\leq i\leq n\\ (\mu_i,M_i)\in\mathfrak{G}_{c_i}}}(\mu_i,M_i)\in\mathfrak{F}_c.$$

Definition: 2.14. [12] Let $(\gamma,K)\in SF_C(X,\tau)$. The pair $((\gamma,K),lim)$ is called as **a soft fuzzy C-convergence space**, where $lim : \mathbb{F}_X((\gamma,K))\to SF_X((\gamma,K))$ provided :

(SFC-C1) $\forall\ \mathfrak{F}_c\in\mathbb{F}_X((\gamma,K)), lim\ \mathfrak{F}_c = \sqcap_{\mathfrak{G}_c\in\mathbb{P}_{(\gamma,K)_m}(\mathfrak{F}_c)} lim\ \mathfrak{G}_c$.

(SFC-C2) $\forall\ \mathfrak{F}_c,\mathfrak{G}_c\in\mathbb{P}_X(\gamma,K)$, when $\mathfrak{F}_c\sqsubseteq\mathfrak{G}_c\Rightarrow lim\ \mathfrak{G}_c\sqsubseteq lim\ \mathfrak{F}_c$.

(SFC-C3) $\forall\ x\in(\gamma,K)_0, 0<\alpha\leq 1, lim\ (\alpha\dot{1}_x,\{\dot{x}\})\sqsupseteq(\alpha 1_x,\{x\})$

(SFC-C4) $\forall\ \mathfrak{F}_c\in\mathbb{P}_X(\gamma,K), lim\ \mathfrak{F}_c\sqsubseteq\mathfrak{c}(\mathfrak{F}_c)$.

A map $f : ((\gamma,K),lim_{(\gamma,K)})\to((\delta,L),lim_{(\delta,L)})$ between two soft fuzzy C-convergence spaces is said to be **soft fuzzy continuous**, when

$$lim_{(\gamma,K)}\mathfrak{F}_c\sqsubseteq lim_{(\delta,L)}f\mathfrak{F}_c$$

for each $\mathfrak{F}_c\in\mathbb{F}_{C_X}((\gamma,K))$.

The category whose objects consists of all soft fuzzy C-convergence spaces and whose morphisms are all the continuous maps between objects is denoted by *SFC-CS*. The class of all objects in *SFC-CS* is denoted by $|$ *SFC-CS* $|$.

Note: Let $((\gamma,K),lim)$ be a soft fuzzy C-convergence space with $(\beta,L)\sqsubseteq(\gamma,K)$. For any soft fuzzy C-filter $\mathfrak{F}_c\in\mathbb{F}_{C_X}((\beta,L))$ $lim|_{(\beta,L)}\mathfrak{F}_c = (\beta,L)\sqcap lim(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)}$. Then, $((\gamma,K),lim|_{(\beta,L)})$ is a soft fuzzy C-convergence subspace.

3 Locally C-precompact and Locally weakly C-precompact spaces:

Definition: 3.1. Let (λ, N) be a soft fuzzy set and $((\gamma, K), lim)$ be an object in $\mid SFC\text{-}CS \mid$. Then, (λ, N) is said to be a ***soft fuzzy C-precompact*** (***soft fuzzy weakly C-precompact***) if $(\lambda, N) \in \mathfrak{F}_c$ implies that $\sqcup_{\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma,K))} lim\ \mathfrak{F}_c \sqsubseteq \mathfrak{c}(\sqcup_{\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma,K))} \mathfrak{F}_c)$ $(\sqcup_{\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma,K))} lim\ \omega\iota\mathfrak{F}_c = (\gamma, K))$ respectively.

Definition: 3.2. A soft fuzzy C-convergence space $((\gamma, K), lim)$ is said to be ***soft fuzzy locally C-precompact space*** (***soft fuzzy locally weakly C-precompact space***) provided that each $\mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma, K))$ contains a soft fuzzy C-precompact (soft fuzzy weakly C-precompact) element respectively.

Proposition: 3.1. Let (X, τ) be a soft fuzzy $F^\sharp$ spaces. Let $((\gamma, K), lim) \in \mid SFC\text{-}CS \mid$. Then, the following are equivalent:

(a) $((\gamma, K), lim)$ is locally C-precompact.

(b) each $\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma, K))$ contains a soft fuzzy C-precompact element.

(c) each $\omega\mathcal{U}$ contains a soft fuzzy C-precompact element when $\mathcal{U}$ is an ultrafilter on $(\gamma, K)_0$.

Proposition: 3.2. Let (X, τ) be a soft fuzzy $F^\sharp$ spaces. Let $((\gamma, K), lim) \in \mid SFC\text{-}CS \mid$. Then, the following are equivalent:

(a) $((\gamma, K), lim)$ is soft fuzzy locally weakly C-precompact.

(b) each $\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma, K))$ contains a soft fuzzy weakly C-precompact element.

(c) each $\omega\mathcal{U}$ contains a soft fuzzy weakly C-precompact element when $\mathcal{U}$ is an ultrafilter on $(\gamma, K)_0$.

Proposition: 3.3. Let (X, τ) be a soft fuzzy $F^\sharp$ spaces. A locally C-precompact object in soft fuzzy C-convergence space is soft fuzzy locally weakly C-precompact convergence space.

Proposition: 3.4. Continuous image of a soft fuzzy C-precompact set is soft fuzzy C-precompact set.

Proof. Let $((\gamma, K), lim_{(\gamma,K)})$ and $((\delta, L), lim_{(\delta,L)})$ be the two objects in soft fuzzy C-convergence space . Assume that $f : ((\gamma, K), lim_{(\gamma,K)}) \to ((\delta, L), lim_{(\delta,L)})$ is soft fuzzy continuous in SFC-CS. Then for all $\mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma, K))$, $lim_{(\gamma,K)}\ \mathfrak{F}_c \sqsubseteq lim_{(\delta,L)}\ f\mathfrak{F}_c$. Let $(\lambda, N) \in \mathfrak{F}_c$ be a soft fuzzy C-precompact set in $((\gamma, K), lim_{(\gamma,K)})$. It must be shown that $f(\lambda, N)$ is a soft fuzzy C-precompact set in $((\delta, L), lim_{(\delta,L)})$. Since $(\lambda, N) \in \mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma, K))$, then, by definition of soft fuzzy C-precompactness, $\sqcup_{\mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma,K))}\ lim_{(\gamma,K)}\ \mathfrak{F}_c \sqsupseteq \mathfrak{c}(\sqcup_{\mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma,K))} \mathfrak{F}_c)$. Now, $f(\lambda, N) \in f\mathfrak{F}_c \in \mathbb{P}_{C_Y}((\delta, L))$. Therefore, $\sqcup_{f\mathfrak{F}_c \in \mathbb{P}_{C_Y}((\delta,L))}\ lim_{(\delta,L)}\ f\mathfrak{F}_c \sqsupseteq \sqcup_{\mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma,K))}\ lim_{(\gamma,K)}\ \mathfrak{F}_c \sqsubseteq \mathfrak{c}(\sqcup_{\mathfrak{F}_c \in \mathbb{P}_{C_X}((\gamma,K))} \mathfrak{F}_c)$. Thus, $f(\lambda, N)$ is a soft fuzzy C-precompact set. □

Proposition: 3.5. Let $((\gamma, K), lim)$ be a soft fuzzy locally C-precompact convergence space and $(\beta, L) \sqsubseteq (\gamma, K)$ be a soft fuzzy closed set in a soft fuzzy C-convergence space. Then, $((\beta, L), lim|_{(\beta,L)})$ is a soft fuzzy locally C-precompact convergence space.

Proposition: 3.6. Let $((\gamma, K), lim)$ be a soft fuzzy C-convergence space and $(\lambda, N), (\mu, M) \sqsubseteq (\gamma, K)$. If $((\lambda, N), lim|_{(\lambda,N)})$ and $((\mu, M), lim|_{(\mu,M)})$ are the soft fuzzy locally C-precompact and $(\eta, S) = (\lambda, N) \sqcup (\mu, M)$, then (η, S) is a soft fuzzy locally C-precompact.

4 On soft fuzzy C-open set $(\lambda, N)^*$:

Let $((\gamma, K), lim_{(\gamma,K)})$ be a non-compact soft fuzzy convergence space. Let

$$N((\gamma, K)) := \{\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma, K)) : lim\mathfrak{F}_c \sqsubset \mathfrak{c}(\mathfrak{F}_c), \text{for} \mathfrak{F}_c \text{is a soft fuzzy prime C-filter}\}$$

and define the equivalence relation $\sim$ on $\mathbb{F}_{C_X}((\gamma,K))$ by $\mathfrak{F}_c \sim \mathfrak{G}_c$ iff $\iota\mathfrak{F}_c = \iota\mathfrak{G}_c$. Let $[\mathfrak{F}_c] := \{\mathfrak{G}_c \in \mathbb{F}_{C_X}((\gamma,K)) : \mathfrak{F}_c \sim \mathfrak{G}_c\}$ and $S((\gamma,K)) := \{[\mathfrak{F}_c] : \mathfrak{F}_c \in N((\gamma,K))\}$. Put $X^* = X \cup S((\gamma,K))$ and for each $[\mathfrak{G}_c] \in S((\gamma,K))$, define the characteristic set, $\mathfrak{c}[\mathfrak{G}_c] = (\vee_{\mathfrak{F}_c\in[\mathfrak{G}_c]} \wedge_{(\lambda,N)\in\mathfrak{F}_c} \vee_{x\in X}\lambda(x), \{[\mathfrak{G}_c]\})$. For $(\mu,M) \sqsubseteq (\gamma,K)$, $(\mu,M)^{\S} = (\mu^{\S}, M^{\S})$ is a soft fuzzy set in X^*, where

$$\mu^{\S}(x) = \begin{cases} \mu(x), & x \in X \\ 0, & x \in S((\gamma,K)) \end{cases}$$

and $M^{\S} = M$. Now, $(\mu,M)^{\dagger} = \sqcup_{\substack{[\mathfrak{G}_c]\in S((\gamma,K)) \\ (\mu,M)\in\mathfrak{F}_c}, \forall\mathfrak{F}_c\in[\mathfrak{G}_c]} (\mathfrak{c}[\mathfrak{G}_c] \sqcap (1_{[\mathfrak{G}_c]}, \{[\mathfrak{G}_c]\}))$. Therefore, $(\mu,M)^* = (\mu,M)^{\S} \veebar (\mu,M)^{\dagger}$. It can be easily seen that the mapping $\S : SF_{C_X}((\gamma,K)) \to SF_{C^*_X}((\gamma,K))$, which embeds $SF_{C_X}((\gamma,K))$ in $SF_{C^*_X}((\gamma,K))$. It is understood that the soft fuzzy C-open set $(\mu,M) \sqsubseteq (\gamma,K)$ becomes as a soft fuzzy subset of X^*.

Let $\mathfrak{F}_c, \mathfrak{G}_c, \mathfrak{H}_c, ... \in \mathbb{F}_{C_X}((\gamma,K))$ and $\Phi_c, \Psi_c, \Lambda_c, ... \in \mathbb{F}_{C_{X^*}}((\gamma,K)^*)$. To a soft fuzzy C-filter $\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma,K))$, there corresponds a soft fuzzy C-filter $\mathfrak{F}^*_c \in \mathbb{F}_{C_{X^*}}((\gamma,K)^*)$ by $\mathfrak{F}^*_c := (\{(\lambda,N)^* : (\lambda,N) \in \mathfrak{F}_c\})^{\boldsymbol{\cdot}}_{(\gamma,K)^*}$ and to a soft fuzzy C-filter $\Phi_c \in \mathbb{F}_{C_{X^*}}((\gamma,K)^*)$, there corresponds a soft fuzzy C-filter, $\widetilde{\Phi}_c = \{(\lambda,N) \sqcap (\gamma,K) \in SF_{C_X}((\gamma,K)) : (\lambda,N)^* \in \Phi_c\}$.

Definition: 4.1. Let X and X^* be any non-empty sets and the soft fuzzy sets A and B be in the form, $A = \{(\mu,M) : \mu(x) \in I^X, \forall x \in X, M \subseteq X\}$ and $B = \{(\lambda,N) : \lambda(x) \in I^{X^*}, \forall x \in X^*, N \subseteq X^*\}$. Then,

(1) $A \barwedge B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X \cap X^*, M \cap N \subseteq X \cap X^*$.

(2) $A \veebar B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X \cup X^*, M \cup N \subseteq X \cup X^*$.

Proposition: 4.1. Let $\mathfrak{F}_c, \mathfrak{G}_c$ be any two soft fuzzy prime C-filters on $(\gamma,K) \in SF_{C_X}((\gamma,K))$. Let $\iota\mathfrak{F}_c = \iota\mathfrak{G}_c$; $(\lambda,N) \sqcup (\mu,M) \in \mathfrak{F}_c$ and $(\lambda,N) \sqcup (\mu,M) \in \mathfrak{G}_c$. Then, $((\lambda,N) \in \mathfrak{F}_c\text{and}(\lambda,N) \in \mathfrak{G}_c)$ or $((\mu,M) \in \mathfrak{F}_c\text{and}(\mu,M) \in \mathfrak{G}_c)$.

Proposition: 4.2. For any soft fuzzy set (γ,K), we have

1. $(0,\phi)^* = (0,\phi)$.
2. $(\sqcup_{i\in I}(\mu,M))^* = \sqcup_{i\in I}(\mu,M)^*$.
3. $((\lambda,N) \sqcap (\mu,M))^* = (\lambda,N)^* \sqcap (\mu,M)^*$
4. $(\lambda,N)(\mu,M) \Rightarrow (\lambda,N)^* \sqsubseteq (\mu,M)^*$
5. $(\lambda,N)^* \barwedge (\gamma,K) = (\lambda,N)$.

Proposition: 4.3. If $\Phi_c \in \mathbb{F}_{C_{X^*}}((\gamma,K)^*)$ is a soft fuzzy prime C-filter, then $\widetilde{\Phi}_c$ is a soft fuzzy prime C-filter.

Proof. Let $(\lambda,N) \sqcup (\mu,M) \in \widetilde{\Phi}_c$ with $\widehat{(\lambda,N)} \neq \phi$ or $\widehat{(\mu,M)} \neq \phi$. Now, $(\lambda,N)^* \sqcup (\mu,M)^* = ((\lambda,N) \sqcup (\mu,M))^* \in \widetilde{\Phi}_c$ with $\widehat{(\lambda,N)^*} \neq \phi$ or $\widehat{(\mu,M)^*} \neq \phi$. Since Φ_c is a soft fuzzy prime C-filter, it follows that $(\lambda,N)^* \in \Phi_c$ or $(\mu,M)^* \in \Phi_c$. This implies that, $(\lambda,N) \in \widetilde{\Phi}_c$ or $(\mu,M) \in \widetilde{\Phi}_c$. □

Proposition: 4.4. Let $\mathfrak{G}_c \in N((\gamma,K))$ and $(0,\phi) \sqsubseteq (\alpha 1_{[\mathfrak{G}_c]}, \{[\mathfrak{G}_c]\}) \sqsubseteq \mathfrak{c}([\mathfrak{G}_c])$. Then, $\iota\mathfrak{G}_c \sqsubseteq \iota\widetilde{(\alpha 1_{[\mathfrak{G}_c]}, \{[\mathfrak{G}_c]\})}^{\boldsymbol{\cdot}}$.

Proposition: 4.5. Let $\Phi_c, \Psi_c \in \mathbb{F}_{C_{X^*}}((\gamma,K)^*)$. Then,

(1) $\Phi_c \sqsubseteq \Psi_c \Rightarrow \widetilde{\Phi}_c \sqsubseteq \widetilde{\Psi}_c$.

(2) $\Phi_c \sqsubseteq \Psi_c \Rightarrow \mathfrak{c}(\Phi_c) \sqsupseteq \mathfrak{c}(\Psi_c)$

(3) $\iota(\Phi_c) = \iota(\Psi_c) \Rightarrow \iota(\widetilde{\Phi}_c) = \iota(\widetilde{\Psi}_c)$.

Proposition: 4.6. Let $\mathfrak{F}_c$ be a soft fuzzy C-filter on $(\beta, L) \sqsubseteq (\gamma, K)$. Then, $\mathbb{P}_{(\gamma,K)_m}((\mathfrak{F}_c)^{\cdot}_{(\gamma,K)}) = \{(\mathfrak{G}_c)^{\cdot}_{(\gamma,K)} : \mathfrak{G}_c \in \mathbb{P}_{(\gamma,K)_m}(\mathfrak{F}_c)\}$.

Proposition: 4.7. Let $\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma, K))$. Then, $\widetilde{(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*}} = \mathfrak{F}_c$.

Proposition: 4.8. Let $\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma, K))$. Then, $\mathbb{P}_{(\gamma,K)_m}(\mathfrak{F}_c) = \{\widetilde{\Phi}_c : \Phi_c \in \mathbb{P}_{(\gamma,K)_m}((\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*})\}$.

Proposition: 4.9. Let $\Phi_c \in \mathbb{F}_{C_{X^*}}((\gamma, K)^*)$. Then, $\widetilde{\Phi}^*_c \sqsubseteq \widetilde{\Phi}_c$. Furthermore, $\mathfrak{G}_c \in \mathbb{F}_{C_X}((\gamma, K))$ such that $\mathfrak{G}^*_c \sqsubseteq \Phi$, then $\mathfrak{G}_c \sqsubseteq \widetilde{\Phi}_c$.

5 Compactification of a soft fuzzy locally C-precompact convergence space:

Let us now define a soft fuzzy C-convergence space on $(\gamma, K)^*$. Define

$$lim^*\Phi_c = (lim\Phi_c)^{\S} \veebar (lim\Phi_c)^{\dagger}$$

Where,
$(lim\Phi_c)^{\S} = lim\widetilde{\Phi}_c \barwedge \mathfrak{c}(\Phi_c)$;
$(lim\Phi_c)^{\dagger} = \mathfrak{c}(\Phi_c) \barwedge (1_{S((\gamma,K))}, S((\gamma, K)))$.

Proposition: 5.1. For a non-locally C-precompact soft fuzzy C-convergence space $((\gamma, K), lim)$, $((\gamma, K)^*, lim^*)$ is also a soft fuzzy C-convergence space.

Proposition: 5.2. For a soft fuzzy non-locally C-precompact convergence space $((\gamma, K), lim)$, the soft fuzzy C-convergence space $((\gamma, K)^*, lim^*)$ is soft fuzzy locally C-precompact space.

Proposition: 5.3. For a soft fuzzy non-locally C-precompact convergence space $((\gamma, K), lim)$, $lim|_{(\gamma,K)} = lim$.

Proof. Let $\mathfrak{F}_c \in \mathbb{F}_{C_X}((\gamma, K))$ be a soft fuzzy prime C-filter. Obviously, $(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*}$ is a soft fuzzy prime C-filter. Now, by the definition of the soft fuzzy C-convergence subspace and by the Proposition: 4.7., $lim^*|_{(\gamma,K)}\mathfrak{F}_c = lim^*(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*} \sqcap (\gamma, K) = \{lim\mathfrak{F}_c \barwedge \mathfrak{c}(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*}\} \veebar \{\mathfrak{c}(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*} \barwedge (1_{S((\gamma,K))}, S((\gamma, K)))\} = lim\ \mathfrak{F}_c \sqcap (\gamma, K) = lim\ \mathfrak{F}_c$. □

Proposition: 5.4. For a soft fuzzy non-locally C-precompact convergence space $((\gamma, K), lim)$, (γ, K) is lim^*-dense set in $((\gamma, K)^*, lim^*)$.

Proof. It is clear that, $\Gamma^{lim^*}(\gamma, K) \sqsubseteq (\gamma, K)^*$. Now, $\Gamma^{lim^*}(\gamma, K) = \sqcup_{(\gamma,K)\in\mathfrak{F}_c} lim^*\ (\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*} = \sqcup_{(\gamma,K)^*\in(\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*}} lim^*\ (\mathfrak{F}_c)^{\cdot}_{(\gamma,K)^*} = \Gamma^{lim}(\gamma, K)^* \sqsupseteq (\gamma, K)^*$. Thus, $\Gamma^{lim^*}(\gamma, K) = (\gamma, K)^*$ □

Proposition: 5.5. For a soft fuzzy non-locally C-precompact convergence space, $((\gamma, K), lim)$ the soft fuzzy C-convergence space $((\gamma, K)^*, lim^*)$ is a soft fuzzy locally C-precompactification.

Proof. From the Propositions (5.1)-(5.4), it is clear that, $((\gamma, K)^*, lim^*)$ is a soft fuzzy locally C-precompactification. □

Proposition: 5.6. (**Tychonoff Theorem:**) Product of any soft fuzzy locally C-precompact space is soft fuzzy locally C-precompact.

Proof. Let $\mathbb{X} = \prod_{j\in J} X_j$ be the product space and $(\mathbb{X}, \tau)$ be the soft fuzzy product topological space. Let $(\gamma, K) = \prod_{j\in J}(\gamma_j, K_j)$ be the product set on $\mathbb{X}$ and each $(\gamma_j, K_j) \in X_j$. Let $\mathfrak{F}_c \in \mathbb{F}_{C_{\mathbb{X}}}((\gamma, K))$ be a soft fuzzy prime C-filter on (γ, K). Then, $P^{(\gamma,K)}_j\mathfrak{F}_c$ is a soft fuzzy prime C-filter on (γ_j, K_j) and $\mathfrak{c}(P^{(\gamma,K)}_j\mathfrak{F}_c) = \mathfrak{c}(\mathfrak{F}_c)$. Since each $((\gamma_j, K_j), lim_{(\gamma_j,K_j)})$ is a soft fuzzy locally C-precompact, then for each $j \in$

Λ, $\sqcup_{P_j^{(\gamma,K)}\mathfrak{F}_c \in \mathbb{F}_{C_{X_i}}((\gamma_i,K_i))} lim_{(\gamma_i,K_i)}\ P_j^{(\gamma,K)}\mathfrak{F}_c \sqsupseteq \mathfrak{c}(\sqcup_{P_j^{(\gamma,K)}\mathfrak{F}_c \in \mathbb{F}_{C_{X_j}}((\gamma_j,K_j))}\ P_j^{(\gamma,K)}\mathfrak{F}_c) = \mathfrak{c}(\sqcup_{\mathfrak{F}_c \in \mathbb{F}_{C_{\mathbb{X}}}((\gamma,K))}\mathfrak{F}_c)$. Now, by the definition of the soft fuzzy locally C-precompactness, $\sqcup_{\mathfrak{F}_c \in \mathbb{F}_{C_{\mathbb{X}}}((\gamma,K))} lim\,\mathfrak{F}_c \sqsupseteq lim\mathfrak{F}_c = \sqcap_{j\in J,\ P_j^{(\gamma,K)}\mathfrak{F}_c \in \mathbb{F}_{C_{X_j}}((\gamma_j,K_j))}$ $lim_{(\gamma_j,K_j)} P_j^{(\gamma,K)}\mathfrak{F}_c \sqsubseteq \mathfrak{c}(\sqcup_{\mathfrak{F}_c \in \mathbb{F}_{C_{\mathbb{X}}}((\gamma,K))}\mathfrak{F}_c)$. This implies that, $((\gamma, K), lim)$ is a soft fuzzy locally C-precompact convergence space. $\square$

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