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LABELING OF SUBDIVIDED GRAPHS

***K.Murugan and A.Subramanian**

Abstract

In this paper we prove that the graph obtained by the subdivision of the edges of the star $S_{1,n}$, the graph obtained by the subdivision of the central edge of the Bistar $B_{m,n}$, the graph obtained by subdividing the edges of the path P_n in P_n □K_1 and P_n □K_2,the graph obtained by subdividing the parallel edges of a H-graph in H□K_1 and in H□K_2 are Skolem difference mean.

Keywords: Star, Bistar, Path, Subdivision, Labeling, Skolem Difference Mean labeling, Skolem Difference Mean graphs.

AMS Subject classification (2010): 05C78

1 Introduction

Throughout this paper, by a graph, we mean a finite, undirected, simple graph. Let G (V, E) be a graph with p vertices and q edges. The Star $S_{1,n}$ is obtained from K_1 by joining n pendant edges with K_1. The Bistar $B_{m,n}$ is the graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2.The edge of K_2 is called the central edge of $B_{m,n}$ and the vertices of K_2 are called the central vertices of $B_{m,n}$. A path is a simple graph obtained from a walk $v_0e_1v_1e_2v_2\ldots e_kv_k$ in which the vertices and edges are distinct. A path on n vertices is denoted by P_n. The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1,v_2,\ldots v_n$ and $u_1,u_2,\ldots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even [3].

* Department of Mathematics, The M.D.T. Hindu College, Tirunelveli–627010, India.
E-mail:murugan_mdt@yahoo.com,asmani1963@gmail.com

A subdivision of a graph G is a graph that can be obtained from G by a sequence of edge subdivisions. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey of many of them can be found in [2]. The concept of Skolem Mean Labeling was introduced in [1].

In this paper, we define Skolem difference mean labeling and discuss the Skolem difference mean labeling of subdivided graphs.

2 Main Results

Definition 2.1 A graph G(V,E) with p vertices and q edges is said to have Skolem Difference Mean labeling if it is possible to label the vertices xεV with distinct elements f(x) from 1,2,3…p+q in such a way that the edge e=uv is labeled with $\frac{|f(u)-f(v)|}{2}$ if |f(u)-f(v)| is even and $\frac{|f(u)-f(v)|+1}{2}$ if |f(u)-f(v)| is odd and the resulting edges get distinct labels from 1,2,3…q. A graph that admits Skolem difference mean labeling is called Skolem difference mean graph.
The Skolem difference mean labeling of the path P_6 is given in figure 1.

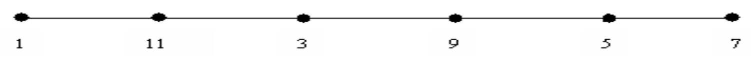

Fig 1

Theorem 2.2 The graph obtained by the subdivision of the edges of the star $S_{1,n}$ is Skolem difference mean for all values of n.

Proof. Let G be the graph obtained by the subdivision of the edges of the star $S_{1,n}$.
Let $V(G)=\{u,u_i,w_i\,;1\le i\le n\}$ and $E(G)=\{uw_i,w_iu_i\,;1\le i\le n\}$
Define the function $f:V(G)\rightarrow\{1,2\ldots 4n+1\}$ by
$f(u)=4n+1$
$f(w_i)=2i-1;1\le i\le n$
$f(u_i)=2n+2-2i;1\le i\le n$
Let f* be the induced edge labeling of f. Then
$f^*(uw_i) = 2n+1-i;\ 1\le i\le n$

$f^*(w_iu_i) = n+2-2i;\ 1\le i\le \frac{n+1}{2}$ and

$=2\,[i-\frac{n+1}{2}];\ \frac{n+3}{2}\le i\le n$ if n is odd

$=n+2-2i;\ 1\le i\le \frac{n}{2}$ and

$=2[i-\frac{n}{2}]-1;\ \frac{n}{2}+1\le i\le n$ if n is even

Then the induced edge labels are 1,2…2n. Hence the theorem.

□

The Skolem difference mean labeling of the graph obtained by the subdivision of the edges of the star $S_{1,4}$ is given in fig 2.

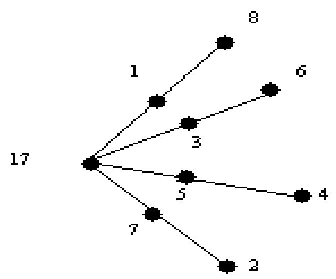

Fig 2

Theorem 2.3 The graph obtained by the subdivision of the central edge of the Bistar $B_{m,n}$ is a Skolem difference mean graph for all values of m and n.

Proof. Let $V(B_{m,n}) =\{u,v,w,u_i,v_j\,;1\le i \le m,\ 1\le j\le n\}$ and $E(B_{m,n}) =\{uw,\ vw,\ uu_i,\ vv_j;1\le i\le m,\ 1\le j \le n\}$ where u and v are the central vertices and let w be the vertex which subdivides the edge uv.
Define the function $f:V(G)\rightarrow\{1,2,3\ldots 2m+2n+5\}$ by
$f(u)=1$,
$f(w)=2m+2n+5$,
$f(v)=3$,
$f(u_i)=2i,1\le i\le m$ and
$f(v_j)=2m+2n+5-2j,1\le j\le n$

Let f* be the induced edge labeling of f. Then
$f^*(uu_i) = i$; $1 \le j \le m$
$f^*(uw) = m+n+2$
$f^*(wv) = m+n+1$
$f^*(vv_j) = m+n+1-j$; $1 \le j \le n$
The induced edge labels are 1,2,3…m+n+2.Hence the theorem.

□

The Skolem difference mean Labeling of the graph obtained by the subdivision of the central edge of the Bistar $B_{6,4}$ is given in fig 3.

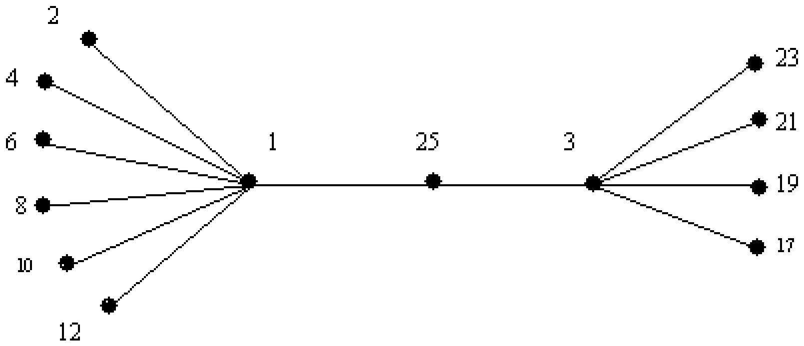

Fig 3

Theorem 2.4 Let G be a graph obtained from the graph $P_n \odot K_1$ by subdividing the edges of the path P_n. Then G is Skolem difference mean for all values of n.

Proof. Let $V(G)=\{u_i,v_i,w_j ;1 \le i \le n, 1 \le j \le n-1\}$ and $E(G)= \{u_iv_i,u_jw_j,w_ju_{j+1} ;1 \le i \le n,1 \le j \le n-1\}$
Define $f: V(G) \to \{1,2…6n-3\}$ by
$f(u_i)=2i-1$, $1 \le i \le n$
$f(v_i)=2n+2-2i$, $1 \le i \le n$
$f(w_j)=6n-1-2j$, $1 \le j \le n-1$
Let f* be the induced edge labeling of f. Then

$f^*(u_iv_i) = n+2-2i$; $1 \le i \le \frac{n+1}{2}$ and

$= 2[\, i-\frac{n+1}{2}]$; $\frac{n+3}{2} \le i \le n$ if n is odd

$=n+2-2i$; $1 \le i \le \frac{n}{2}$ and

$=2[i-\frac{n}{2}]-1$; $\frac{n}{2}+1 \le i \le n$ if n is even.

$f^*(u_jw_j) = 3n-2j$; $1 \le j \le n-1$
$f^*(w_ju_{j+1}) = 3n-1-2j$; $1 \le j \le n-1$
Then the induced edge labels are 1,2…3n-2.Hence the graph G is Skolem difference mean for all n.

□

The Skolem difference mean Labeling of the graph obtained by subdividing the edges of the path P_5 of the graph $P_5 \odot K_1$ is given in fig 4.

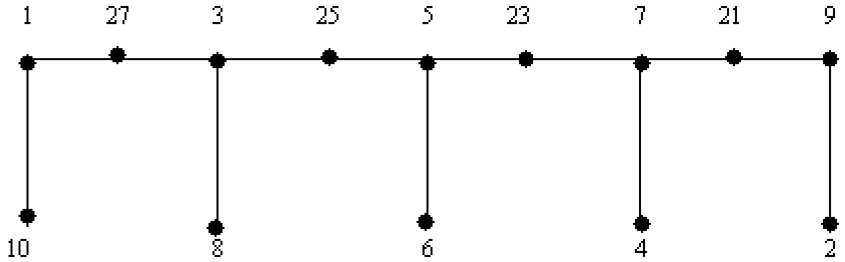

Fig 4

Theorem 2.5 Let G be a graph obtained from the graph P_n □ K_2 by subdividing the edges of the path P_n. Then G is Skolem difference mean for all values of n.

Proof. Let $V(G)=\{u_i, v_{i1}, v_{i2}, w_j; 1\leq i\leq n, 1\leq j\leq n-1\}$ and

$E(G)= \{u_iv_{i1}, u_iv_{i2}, u_jw_j, w_ju_{j+1}; 1\leq i\leq n, 1\leq j\leq n-1\}$

Define f: $V(G)\rightarrow\{1,2\ldots 8n-3\}$ by

$f(u_i)=2i-1,\ 1\leq i\leq n$

$f(v_{i1})=6i-4,\ 1\leq i\leq n$

$f(v_{i2})=6i-2,\ 1\leq i\leq n$

$f(w_j)=8n-1-2j,\ 1\leq j\leq n-1$

Let f* be the induced edge labeling of f. Then

$f^*(u_iv_{i1}) = 2i-1;\ 1\leq i\leq n$

$f^*(u_iv_{i2}) =2i;\ 1\leq i\leq n$

$f^*(u_jw_j) = 4n-2j;\ 1\leq j\leq n-1$

$f^*(w_ju_{j+1}) = 4n-1-2j;\ 1\leq j\leq n-1$

Then the induced edge labels are 1,2…4n-2.Hence the graph G is Skolem difference mean for all n.

□

The Skolem difference mean Labeling of the graph obtained by subdividing the edges of the path P_5 of the graph P_5□K_2 is given in fig 5.

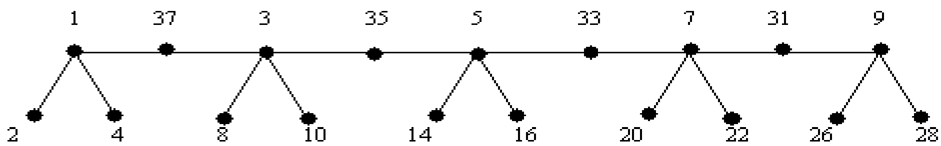

Fig 5

Theorem 2.6 The graph G obtained by the subdivision of the parallel edges of a H-graph in H□K_1 is Skolem difference mean.

Proof. Let $V(G)=\{u_i, u_i', v_i, v_i', w_j, t_j; 1\leq i\leq n, 1\leq j\leq n-1\}$ and

$$E(G)=\{u_iu_i', v_iv_i', u_jw_j, w_ju_{j+1}, v_jt_j, t_jv_{j+1} \text{ and } v_{\frac{n+1}{2}}u_{\frac{n+1}{2}} \text{ if n is odd\& } v_{\frac{n}{2}+1}u_{\frac{n}{2}} \text{ if n is even}; 1\leq i\leq n, 1\leq j\leq n-1\}$$

Define a function $f: V(G)\rightarrow\{1,2\ldots 12n-5\}$ by

$f(u_i) = 2i-1;\quad 1\leq i\leq n$

$f(v_i) = 10n-1-2i;\ 1\leq i\leq n$

$f(u_i') = 4n+2-2i;\ 1\leq i\leq n$

$f(v_i') = 6n-2+2i;\ 1\leq i\leq n$

$f(w_j) = 12n-3-2j; 1\leq i\leq n-1$

$f(t_j) = 2n-1+2i;\quad 1\leq i\leq n-1$
Let f* be the induced edge labeling of f. Then
$f^*(u_iu_i') = 2n+2-2i;\ 1\leq i\leq n$
$f^*(v_i,v_i') = 2n+1-2i;\ 1\leq i\leq n$
$f^*(u_jw_j) = 6n-1-2j;\ 1\leq j\leq n-1$

$f^*(w_ju_{j+1}) = 6n-2-2j;\ 1\leq j\leq n-1$
$f^*(v_jt_j) = 4n-2j;\ 1\leq j\leq n-1$
$f^*(t_jv_{j+1}) = 4n-1-2j;\ 1\leq j\leq n-1$

$f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) = f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) = 4n-1$

Then the induced edge labels are1,2…6n-3.Hence the theorem. □

The Skolem difference mean labeling of a graph G obtained by the subdivision of the parallel edges of a H-graph in $H\square K_1$ is given in figures 6 and 7.

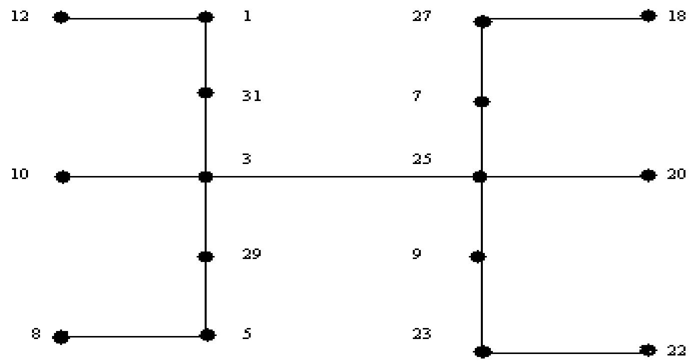

Fig 6

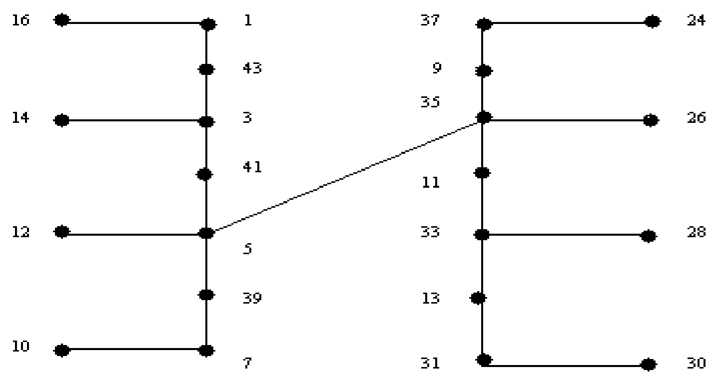

Fig 7

Theorem 2.7 The graph G obtained by the subdivision of the parallel edges of a H-graph in $H\square K_2$ is Skolem difference mean.

Proof. Let $V(G)=\{u_i,u_i',u_i'',v_i,v_i',v_i'',w_j,t_j;1\leq i\leq n,1\leq j\leq n-1\}$ and

$E(G)=\{u_iu_i', u_iu_i'', v_iv_i', v_iv_i'',u_jw_j,w_ju_{j+1},v_jt_j, t_jv_{j+1}$ and $v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}$ if n is odd &

$v_{\frac{n}{2}+1} u_{\frac{n}{2}}$ if n is even;1≤i≤n,1≤j≤n-1}

Define a function f:V(G)→{1,2…16n-5}by
$f(u_i)=2i-1$; 1≤i≤n
$f(u_i')=8n+7-6i$; 1≤i≤n
$f(u_i'')=8n+5-6i$; 1≤i≤n
$f(v_i)= 14n-1-2i$; 1≤i≤n
$f(v_i')=6n-4+6i$; 1≤i≤n
$f(v_i'')=6n-2+6i$; 1≤i≤n
$f(w_j)=16n-3-2j$;1≤j≤n-1
$f(t_j)=2n-1+2j$; 1≤j≤n-1
Let f* be the induced edge labeling of f. Then
$f^*(u_iu_i') = 4n+4-4i$; 1≤i≤n
$f^*(u_iu_i'') = 4n+3-4i$; 1≤i≤n
$f^*(v_iv_i') = 4n+2-4i$; 1≤i≤n
$f^*(v_iv_i'') = 4n+1-4i$; 1≤i≤n
$f^*(u_jw_j) = 8n-1-2j$; 1≤j≤n-1
$f^*(w_ju_{j+1}) = 8n-2-2j$; 1≤j≤n-1
$f^*(v_jt_j) = 6n-2j$; 1≤j≤n-1
$f^*(t_jv_{j+1}) = 6n-1-2j$; 1≤j≤n-1

$f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) = f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) = 6n-1$

Then the induced edge labels are1,2…8n-3.Hence the theorem. □

The Skolem difference mean labeling of a graph G obtained by the subdivision of the parallel edges of a H-graph in H□K_2 is given in figures 8 and 9.

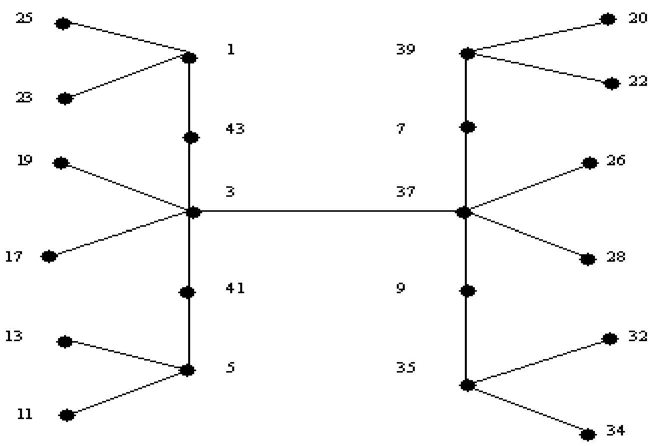

Fig 8

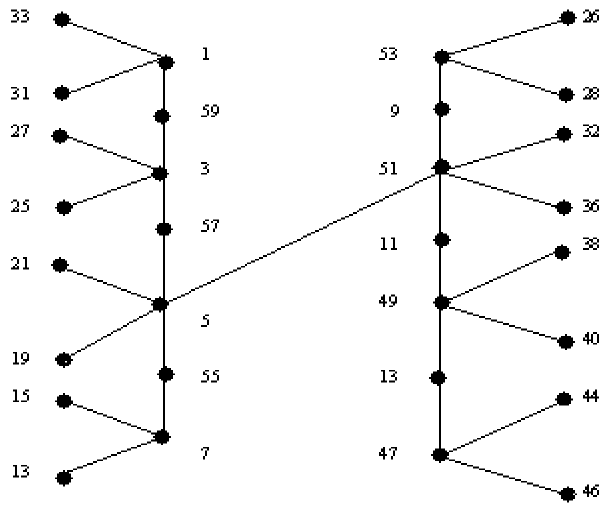

Fig 9

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