

## Design and Implementation of Fractional order controllers for DC Motor Position servo system

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### Abstract

*In this paper a controller is designed for the DC motor servo system using fractional methods. A simple fractional PI controller is designed based on Zeigler Nicols type of rules. Conventional integer order PI controllers can be designed using the tuning rules proposed based on Zeigler Nicols or Modified Zeigler Nicols method. But to design a fractional order PI controller there no such standard methods available for design. In addition to this the Zeigler Nicols type of rules are applicable only to system modeled as first order plus delay time systems. The DC motor, which is a second order system, can be approximated as a first order plus time delay system. We use Zeigler Nicols type of rules to design fractional order controller for the DC motor system. We designed and analyzed the performance of the DC motor system for this fractional order system. It is found that the performance of the fractional order system is better than the Integer order controller.*

**Keywords:** DC motor or servo system, fractional order controller, Zeiger-Nocols tuning rule.

### 1. Introduction

In recent years, the theory and applications of fractional order calculus attracts more researchers towards the applications of fractional order calculus in the field of engineering especially in control systems. The systems which are earlier modeled as distributed parameter systems can easily and more accurately be modeled as the fractional order systems. The analysis of these fractional order systems can be done using fractional calculus. The systems which are modeled as integer order systems can enjoy the advantage of fractional calculus by employing fractional order controllers. The controllers designed making use of fractional calculus could achieve better performance and robustness over the conventional integer order controllers.

Integer order PI controllers can be designed based on Zeigler-Nicols criterion. This criterion is applicable for time delay systems. So these rules are widely used in process industries. The same rules can be applied to DC motor system also by approximating the DC motor transfer function as first order plus time delay systems. The performance of the closed loop systems will be quiet satisfactory if we use integer order PI controllers for the control purpose. But the PI controller parameters are high for this integer type of controllers.

In this paper we propose to use fractional order PI controller for the control of DC motor speed. The Zeigler-Nicols type of proposed in [1] cannot be used directly for the DC motor system. This rule is developed for the time delay system. We convert the motor model which is a second order system into a first order plus time delay system. Then we use the proposed method for the design of fractional order PI controller for this reduced order model. The proposed controller is analyzed for its performance using MATLAB m-file programming. The performance of the Integer order PI controller and fractional order PI controller are compared and it is found that the fractional order PI controller is best in terms of time response analysis. The rest of this paper is organized as follows: In section 2 we give brief introduction about the fractional order controller. In section 3 we discuss the DC motor position servo systems and its modeling. In section 4 we describe the fractional order controller design for the DC motor position servo system. In section 5 we analyze the performances of the integer order and fractional order controller employed for the DC motor system. Finally conclusions are made in Section 6.

### 2. Fractional order controller

The concept of fractional order controller means the controller which can be described by using fractional order calculus. A commonly used definition of the fractional calculus is the Riemann-Liouville

definition which is given in [5]. The most common form of a fractional order PID controller is the  $PI^\lambda D^\mu$ , involving an integrator of order  $\lambda$  and a differentiator of order  $\mu$ . These orders can be real numbers as against the conventional integers. The transfer function of such a controller can be written as

$$G_c(s) = K_p + K_I/s^\lambda + K_D s^\mu$$

Where  $\lambda, \mu > 0$

The control signal  $u(t)$  can be expressed in the time domain as

$$u(t) = K_p e(t) + K_I D^{-\lambda} e(t) + K_D D^\mu e(t)$$

If  $\lambda = 1, \mu = 1$ , the controller is a integer order PID controller.

If  $\lambda = 1, \mu = 0$  the controller is a integer order PI controller.

If  $\lambda > 1, \mu = 0$ , the controller is a fractional order PI controller.

The classical type of PID controllers are special type of fractional order controllers with  $\lambda = 1, \mu = 1$ . It is always expected that the fractional PID controller  $PI^\lambda D^\mu$  may enhance the system performance and provide better control of dynamic systems. The fractional PID controllers are more robust, i.e. the performance of the controller is least affected by the change in the parameters of the perturbed system. The fractional order controller provides two more parameters to adjust the dynamics of the system and hence two more degrees of freedom. The dynamics of the system can be adjusted properly using fractional order systems.

Implementing the fractional order controllers using MATLAB is an important implementation issue. MATLAB doesn't support the implementation of fractional derivatives and integrals. The fractional toolbox proposed in [5] can be used to implement the fractional parameters using MATLAB m-file. Further the fractional order controller can also be implemented in hardware and is given in [6].

### 3. DC servo motor

The dynamic equations and the open-loop transfer function of the DC Motor are [8]:

$$\begin{aligned} s(Js + b)\Theta(s) &= KI(s) \\ (Ls + R)I(s) &= V - Ks\Theta(s) \\ \frac{\Theta}{V} &= \frac{K}{(Js+b)(Ls+R)+K^2} \end{aligned}$$

And the system schematic under open loop and closed loop looks like:

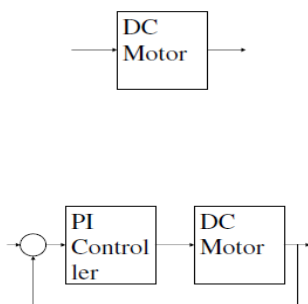


Figure 1: Open loop and closed loop system

With a 1 rad/sec step input, the design criteria are: Settling time, Overshoot and Steady-stage error

The nominal parameters are chosen as [8].  $J=0.01$ ;  $b=0.1$ ;  $K=0.01$ ;  $R=1$ ;  $L=0.5$ ;

The transfer function is

$$\frac{K}{[(J * L)((J * R)s^2 + (L * b))((b * R)s + K2)];}$$

The open loop system is modeled as a transfer function. The time response for this system for a step input signal may produce the response but it may not be in the required form. The open loop response for the system is given in fig (1):

From the open loop response it is found that the system response is not reaching the final steady state also. It is having a permanent steady state error. The introduction of any type of controller only will rectify this problem.

$$K_p = \frac{0.2978}{K(\tau + 0.000307)}$$

$$K_i = \frac{K_p(\tau^2 - 3.402\tau + 2.405)}{0.8578T}$$

$$\alpha = \begin{pmatrix} 0.7, \text{if } \tau < 0.1 \\ 0.9, \text{if } 0.1 < \tau < 0.4 \\ 1.0, \text{if } 0.4 < \tau < 0.6 \\ 1.1, \text{if } \tau > 0.6 \end{pmatrix}$$

#### 4. Design and implementation of fractional order controller

Lots of work on fractional controller is found in literature [1,2,7]. An algorithm is proposed in [1] for the design of Fractional order PI controller for a water level control system.

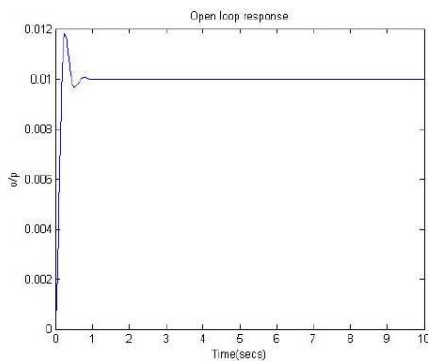


Figure 2. DC motor response to step input under open l  $C(s) = K_p + K_i s^\alpha$

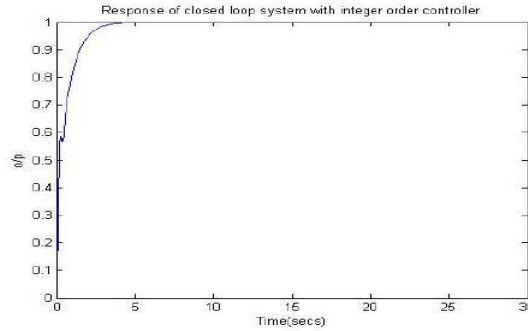
They developed equations for the design of  $K_p$ ,  $k_i$  and  $\alpha$  of fractional order PI controller. The fractional order PI controller is written as Where  $K_p$  and  $K_i$  are proportional and integral constants.  $\alpha$  is the non-integer order of the fractional order integrator. The tuning rules developed in [1] are

These tuning rules are based on fractional Ms constrained integral gain optimization method for generic first order plus delay time model given by

$$G(s) = \frac{Ke^{-Ls}}{(Ts+1)}$$

and

$$\tau = \frac{L}{L+T}$$



**Figure 3. Closed loop Response of DC motor with integer order controller**

is the relative delay. These tuning rules proposed here in cite depends only on the value of  $\tau$ .

The DC motor system is modeled as the second order system. To use this method of design of fractional order controller, it is necessary to model the motor as first order plus time delay system i.e. to determine the values of  $K$ ,  $L$  and  $T$ . Once these values are obtained the values of fractional parameters can be designed using the equations listed above.

### 5. Simulation Results

The fractional order PI controller is designed and implemented for a DC position servo system. First an approximate

First order plus delay time model of the system is obtained. The values of the designed parameters are  $K=0.099$ ,  $L=0.0901$ ,  $T=0.5093$ . Based on these values the transfer function is written as

$$G(s) = \frac{0.099e^{-0.0901s}}{(0.5093s+1)}$$

We design the Integer order control system using Zeigler Nicols criterion. The design formulae is given by

$$K_p = \frac{0.9T}{KL}$$

$$K_i = \frac{K_p}{3L}$$

The values of  $K_p$  and  $K_i$  obtained are 50.922 and 188.3995. The closed loop system is simulated in MATLAB m-file and the step response is obtained. The figures fig(2) and fig(3) shown below gives the response of the closed loop system with integer order controller.

A fractional order PI controller is designed for the same system and the values are given as  $K_p = 19.7896$ ,  $K_i = 86.7998$  and  $a = 0.9$ . It is found that the values of  $K_p$  and  $K_i$  are very high for the integer order controller and it is just half in the case of fractional order controller. The closed loop response of the system with fractional PI controller is given in fig (4) and fig (5).

It is found that the performances of the closed loop system with integer and fractional order controllers are different in steady state and transient state. It is found that the steady state response is good for the integer order controller whereas the transient response is very good for the fractional order controller. In addition to this the gain parameters such as proportional gain and integral gain are high for integer order controllers and they take smaller values in case of fractional order controller. These values are very important and pose some problems during hardware realization.

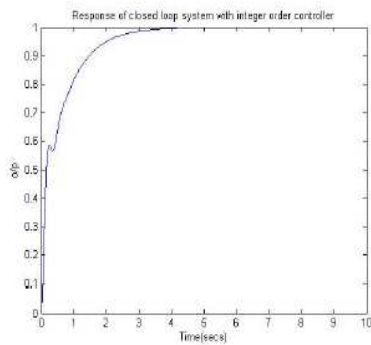


Figure 4. Closed loop Response of DC motor with integer order controller

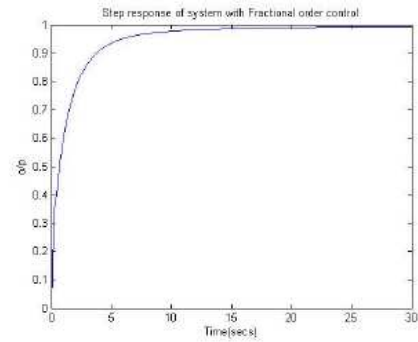


Figure 5. DC motor response to step input with Fractional PI controller

6.

## Conclusions

In this paper we designed a fractional order PI controller for a DC position servo system and implemented it using MATLAB m-file programming. We also implemented the integer order PI controller for the same system. We obtained the closed loop step response for both the systems. We compared the performances of these systems using both integer and fractional order controllers. It is found that the transient performance of the fractional order controller is very good and the steady state performance is good in case of integer order controller. It is also found that the fractional order controller parameters are very small when compared to the integer order controller parameters. This may be very helpful in implementing the controller in hardware.

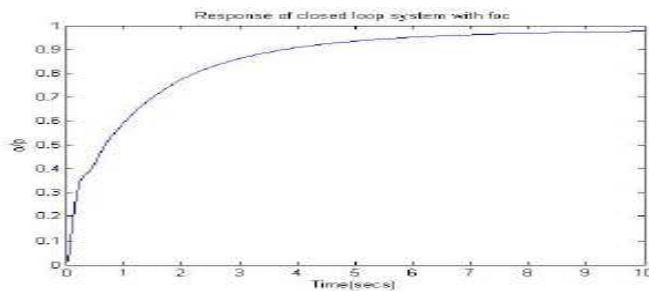


Figure 6. DC motor response to step input with Fractional PI controller

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