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A Common FIXED POINT THEOREM for weakly compatible mappings in 2-metric Spaces

RITU ARORA
Gurukula Kangri Vishwavidyalaya Hardwar (Uttarakhand),

ABSTRACT

In 1982, S. Seesa an Italian mathematician was introduced weak commutativity a pair of maps in fixed point considerations. Thereafter a number of generalizations of this notion have been obtained. Gahler introduced the concept of 2-metric spaces and Iseki for the first time established a fixed point theorem in 2-metric space, since then a number of authors have studied the aspects of fixed point theory in the setting of 2-metric space. Especially, Murthy-Chang-Cho introduced the concepts of compatible mappings and proved coincidence point theorems and common fixed point theorem for these mappings in 2-metric space.

The purpose of this paper this paper is to obtained common fixed point theorem of weakly compatible mappings in 2-metric space.

Key words and phrases: Weakly commuting maps, compatible maps.

Introduction

In 1976, Jungck proved a common fixed point theorem. This Theorem has many applications, but suffers from one drawback that the definitions requires T be continuous throughout X. There then follows a flood of papers involving contractive definition that do not require the continuity of T. This result was first generalized and extended the various ways by many authors.

On the other hand Sessa defined weak commutativity, the so called compatibility which is more general than that of weak commutativity. Since then various fixed point theorems, for compatible mappings satisfying contractive type condition and assuming continuity of at least one of the mappings have been obtained by many authors.

It has been from the paper of kannan that there exist maps that have a discontinuity in the domain but which have fixed points, moreover, the maps involved in every case were continuous at the fixed point. In 1998, Jungck and

Rhoades introduced the notion of weakly compatible and showed that compatible maps are weakly compatible but converse is not true.

Gähler introduced the concept of 2-metric spaces. A 2-metric space is a set X with function $d: XxXxX \to [0, \infty)$ satisfying the following conditions:

(G_1) $for\ two\ distinct\ points\ x, y\ \in X, \exists\ a\ point\ z \epsilon X\ such\ that\ d(x, y, z) \neq 0$

(G_2) $d(x, y, z) = 0\ if\ atleast\ two\ of\ x, y, z\ are\ equal$

(G_3) $d(x, y, z) = d(x, z, y) = d(y, z, x)$

(G_4) $d(x, y, z) \leq d(x, y, u) + d(x, u, z) + d(u, y, z) for\ all\ x, y, z, u = X.$

It has been shown by Gähler that in a 2-metric d is continuous function of any one of three arguments but it is continuous in two arguments. Then it is continuous in all three arguments. A 2-metric d which is continuous in all of its arguments will be called continuous.

Iséki for the first time established a fixed point theorem in 2-metric spaces. Since then a quite numbers of authors have extended and generalizations the result of Iséki and various other results involving contractive and expansive type mappings. Especially, Murty et al. introduced the concept of compatible mapping of Type (A) is 2-metric space, derived some relations between these mappings and proved common fixed point theorems for compatible mappings of type (A) in 2-metric spaces.

On the other hand, Cho, constantin, khan - fisher and kubiak established some necessary and sufficient condition which guarantee the existence of common fixed point for a pair of continuous mappings in 2-metric spaces.

In the last four decades, a number of authors have studied the aspects of fixed point theory in the setting of 2-metric spaces. Especially introduced the concept of compatible mapping, compatible mapping of type (A), compatible mappings of type (P) derived some relations among these mappings and proved coincidence point theorems and common fixed point theorems for these mappings in 2-metric spaces.

In this paper we established the existence of common fixed point of a pair of mappings in 2-metric spaces and obtain coincidence point and common fixed point theorem for weakly compatible mappings.

Preliminaries

Throughout this paper, (Y, d) denotes 2-metric spaces, N and ω denotes the set of positive and nonnegative integers respectively.

Let $R^+ = [0, \infty)$ and

$\phi_1 = \{\phi : \phi : (R^+)^5 \to R^+$ satisfies conditions (a_1) and (a_2) },

$\phi_2 = \{\phi : \phi : (R^+)^{11} \to R^+$ satisfies conditions (a_1) and (a_3) },

Where conditions (a_1), (a_2) and (a_3) are as follows -

(a_1) ϕ is an upper semi continuous, nondecreasing in each coordinate variable.

(a_2) $b(t) = \max \{\phi(t, 0, 0, t, t), \phi(t, t, t, 2t, 0), \phi(t, t, t, 0, 2t)\} < t$ for all $t > 0$

(a_3) $c(t) = \max \{\phi(t, t, t, 0, 2t, t, 0, 2t, 0, 2t, 0), \phi(t, 0, 0, t, t, 0, 0, 0, 0, 0, t),$

$\phi(t, t, t, 2t, 0, t, 2t, 0, 2t, 0, 0)\}, < t$ for all $t > 0$.

LEEMA 2.1. For every $t > 0$, $c(t) < t$ if and only if $\lim_{n\to\infty} c^n(t) = 0$, > 0, where c^n denotes the n times composition of c.

Definition 2.1 (Jungck[11]). Let P and S be mappings from a 2-metric space (Y, d) into itself. P and S are said to be compatible if $\lim_{n\to\infty}(PSx_n,\ SPx_n, a) = 0$ for all $a \in Y$, whenever $\{x_n\}_{n\in N} \subset Y$ such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = t$

Definition 2.2 (Jungck & Rohadas[13]). Let P and S be mappings from a 2-metric space(Y, d) into itself. P and S are said to be weakly compatible if they are commuting on their coincidence point, $Psx = SPx \text{ where } Px = Sx$

Fixed Point Theorem

Theorem 3.1 Let S and T be two mappings from a complete 2-metric space (Y, d) into itself. Then the following conditions are equivalent.

(3.1) S and T have common fixed point.

(3.2) $\exists\, r \in (0, 1)$, $P : Y \to T(Y)$ and $Q : Y \to S(Y)$ such hat

(b_1) the pairs (P, S) and (Q, T) are weakly compatible.

(b_2) One of P, Q, S ant T is continuous.

(b_3) $d(Px, Qy, a) \le r \max d(Sx, Ty, a), d(Sx, Px, a), d(Ty, Qy, a),$

$1/2[d(Sx, Qy, a) + d(Ty, Px, a)]$ for all $x, y, a \in Y$;

(3.3) $\exists\, \phi \in \phi_1$, $P : Y \to T(Y)$ anda $Q : Y \to S(Y)$ satisfying conditions (b_1), (b_2) and (b_4)

(b_4) $d(Px, Qy, a) \le \phi(d(Sx, Ty, a), d(Sx, Px, a), d(Ty, Qy, a), d(Px, Qy, a)$

$+ d(Ty, Px, a))$ for all $x, y, a \in Y$

(3.4) $\exists\, \phi \in \phi_2$, $P : Y \to T(Y)$ and $Q : Y \to S(Y)$ satisfying condition (b_1) (b_2) and (b_5)Sa(

(b_5)Sa $d^2(Px, Qy, a) \le \phi(d^2(Sx, Ty, a),$

$d(Sx, Ty, a)d(Sx, Px, a), d(Sx, Ty, a)d(Ty, Qy, a),$

$d(Sx, Ty,a)d(Sx, Qy, a), d(Px, Ty,a)d(Ty, Px, a),$

$d(Sx, Px, a)d(Ty, Qy, a), d(Sx, Px, a)d(Sx, Qy, a),$

$d(Sx, Px, a)d(Ty, Px, a), d(Ty, Px, a)d(Ty, Qy, a),$

$d(Sx, Qy, a)d(Ty, Qy, a), d(Ty, Px, a) d(Ty, Px, a),$

$d(Sx, Qy, a)d(Ty,Px,a))$ for all $x, y, a \in Y$;

(3.1) $\Rightarrow$ (3.2) and (3.4). Let z be common fixed point of S and T. Define

$P : Y \rightarrow T(Y)$ and $Q : Y \rightarrow S(Y)$ by $Px = Qx = z$ for all $x \in Y$. Then (b_1) and

(b_2) holds. For each $r \in (0, 1)$ and $\phi \in \phi_2$, (b_3) and (b_5) also hold.

(3.2) $\Rightarrow$ (3.3). Take $((\phi, u, v, w, x, y) = r \max \{u, v, w, ½ (x + y)\}$ for all $u, v, w, x, y \in R^+$ .Then $\phi \in \phi_1$ and (b_3) $\Rightarrow$ (b_4)

(3.3) $\Rightarrow$ (3.1). By using this method of Cho[, we can similarly show that (3.4) $\Rightarrow$ (3.1).

Let x_0 be an arbitrary point in Y since $P(Y) \subset T(Y)$ and $Q(Y) \subset S(Y)$ there exist sequence $\{x_n\}_{n\in\omega}$ and $\{y_n\}_{n\in\omega}$ in Y.

$$Px_{2n} = Tx_{2n+1} = z_{2n} \text{ and } Qx_{2n+1} = Tx_{2n+2} = z_{2n+1}$$

Then form the papers of Tan Liu and kim and Lio Zhang and Mao show that $\{y_n\}_{n\in\omega}$ is a Cauchy sequence. It follows from the completeness of (Y, d) that $\{y_n\}_{n\in\omega}$ converges to a point $u \in Y$.

Now suppose that T is continuous. Since the pair (Q, T) are weakly compatible and $\{Qx_{2n+1}\}_{n\in\omega}$ and $\{Tx_{2n+1}\}_{n\in\omega}$ converges to a point u, such that

$QTx_{2n+1} , TQx_{2n+1} \rightarrow u, n \rightarrow \infty$

In virtue of (b_5), we have

$$\begin{aligned}
d^2 (Px_{2n}, QTx_{2n+1}, a) \le \phi\, (&d^2(Sx_{2n}, TQx_{2n+1}, a),\\
&d(Sx_{2n}, TQx_{2n+1}, a)d(Sx_{2n}, Px_{2n}, a),\\
&d(Sx_{2n},TQx_{2n+1}, a)d(TQx_{2n+1}, QTx_{2n+1}, a),\\
&d(Sx_{2n}, TQx_{2n+1}, a)d(Sx_{2n}, QTx_{2n+1}, a),\\
&d(Sx_{2n}, TQx_{2n+1}, a)d(TQx_{2n+1}, Px_{2n}, a),\\
&d(Sx_{2n}, Px_{2n}, a) d(TQx_{2n+1}, QTx_{2n+1}, a),\\
&d(Sx_{2n}, Px_{2n+1}, a)d(Sx_{2n}, QTx_{2n+1}, a),\\
&d(TQx_{2n+1}, QTx_{2n+1}, a)d(Sx_{2n}, QTx_{2n+1}, a),\\
&d(TQx_{2n+1}, QTx_{2n+1},a)d(TQx_{2n+1}, Px_{2n}, a),\\
&d(Sx_{2n}, QTx_{2n+1}, a) d(TQx_{2n+1}, Px_{2n}, a),
\end{aligned}$$

Letting $n \rightarrow \infty$, we have

$$\begin{aligned}
d^2(u, Tu, a) &\le \phi(d^2 (u, Tu, a), 0, 0, d^2 (u, Tu, a), d^2 (u, Tu, a), 0, 0, 0, 0, 0, d^2 (u, Tu, a)).\\
&\le c(d^2(u, Tu, a)).
\end{aligned}$$

Which implies that $u = Tu$. It follows from (b_5) that

$d^2(Px_{2n}, Qu, a) \le \phi\, (d^2(Sx_{2n}, Tu, a), d(Sx_{2n}, Tu, a)d(Sx_{2n}, Qu, a),$

$d(Sx_{2n}, Tu, a)d(Tu, Qu, a), d(Sx_{2n}, Tu, a)d(Sx_{2n}, Qu, a),$

$d(Sx_{2n}, Px_{2n}, a)d(Tu, Qu, a), d(Sx_{2n}, Px_{2n}, a)d(Tu, Px_{2n}, a),$

$d(Tu, Qu, a)d(Sx_{2n}, Qu, a), d(Tu, Qu, a)d(Tu, Px_{2n}, a),$

$d(Tu, Qu, a) d(Tu, Px_{2n}, a), d(Sx_{2n}, Qu, a)d(Tu, Px_{2n}, a),$

As $n \to \infty$, we have

$d^2(u, Qu, a) \leq \phi (d^2(u, Qu, a), 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

$\leq c (d^2(u, Qu, a)),$

This gives that u = Qu. It follows from $Q(Y) \subset S(Y)$,that there exist $v \in Y$, which

u = Qu = Sv. From (b_5), we get

$d^2(Pv, u, a) = d^2(Pv, Qu, a)$

$\leq (d^2(Sv, Tu, a), (Sv, Tu, a)d(Sv, Pv, a),$

$d(Sv, Tu, a)d(Tu, Qu, a), d(Sv, Tu, a)d(Sv,Qu, a),$

$d(Sv, Tu, a)d(Tu, Pv a), d(Sv, Pv, a)d(Tv, Qu, a),$

$d(Sv, Pv, a)d(Sv, Qu, a), d(Sv, Pv, a)d(Tu, Pv, a),$

$d(Tu, Qu, a)d(Sv, Qu, a), d(Tu, Qu, a)d(Tu, Pv, a),$

$d(Sv, Qu, a)d(Tu, Pv, a),$

$= \phi (0, 0, 0, 0, 0, 0, 0, d^2(Pv, u, a), 0, 0, 0)$

$\leq c (d^2(Pv, u, a)).$

Therefore u = Pv. By (b_5) , we obtain again

$d^2(Pu, u, a) = d^2(Pu, Qu, a)$

$\leq \phi(d^2(Su, Tu, a), (Su, Tu, a)d(Su, Pu, a),$

$d(Su, Tu, a)d(Tu, Qu, a), d(Su, Tu, a)d(Su,Qu, a),$

$d(Su, Tu, a)d(Tu, Pu a), d(Su, Pu, a)d(Tu, Qu, a),$

$d(Su, Pu, a)d(Su, Qu, a), d(Su, Pu, a)d(Tu, Pu, a),$

$d(Tu, Qu, a)d(Su, Qu, a), d(Tu, Qu, a)d(Tu, Pu, a),$

$d(Su, Qu, a)d(Tu, Pu, a),$

$\leq \phi(d^2(Pu, u, a), 0, 0, (d^2(Pu, u, a), (d^2(Pu, u, a),$

$0, 0, 0, 0, 0, (d^2(Pu, u, a)),$

$\leq c (d^2(Pu, u, a)).$

Hence u = Pu. That is, u is a common fixed point of P, Q, S and T.

Suppose that P is continuous. Since P and S are weakly compatible and $\{Px_{2n}\}_{n\in\omega}$ and $\{Sx_{2n}\}_{n\in\omega}$ converges to the point u and PSx_{2n} , $SPx_{2n} \to Pu$ as $n \to \infty$. From (b_5) we have

$d^2(PSx_{2n}, Qx_{2n+1}, a) \leq \phi (d^2(SPx_{2n}, Tx_{2n+1}, a),$

$d(SPx_{2n}, Tx_{2n+1}, a)d(SPx_{2n}, PSx_{2n}, a),$

$d(SPx_{2n}, Tx_{2n+1}, a)d(Tx_{2n+1}, Qx_{2n+1}, a),$

$d(SPx_{2n}, Tx_{2n+1}, a)d(Tx_{2n+1}, PSx_{2n}, a),$

$d(SPx_{2n}, PSx_{2n}, a)d(Tx_{2n+1}, Qx_{2n+1}, a),$

$d(SPx_{2n}, PSx_{2n}, a)d(SPx_{2n}, Qx_{2n+1}, a),$

$d(SPx_{2n}, PSx_{2n}, a)d(Tx_{2n+1}, PSx_{2n+1}, a),$

$d(Tx_{2n+1}, Qx_{2n+1}, a)d(SPx_{2n}, Qx_{2n+1}, a),$

$d(Tx_{2n+1}, Qx_{2n+1}, a)d(Tx_{2n+1}, PSx_{2n}, a),$

$d(SPx_{2n}, Qx_{2n+1}, a)d(Tx_{2n+1}, PSx_{2n}, a),$

Letting $n \to \infty$, we get that

$d^2(Pu, u, a) \le \phi(d^2(Pu, u, a), 0, 0, d^2(Pu, u, a), d^2(Pu, u, a), 0, 0, 0, 0, 0, d^2(Pu, u, a)).$

$\le c(d^2(Pu, u, a)).$

This gives that u = pu. Note that $P(Y) \subset T(Y)$. thus there exists with u = Tv. It follows from (b_5) that

$d^2(Px_{2n}, Qv, a) \le \phi(d^2(Sx_{2n}, Tv, a), d(Sx_{2n}, Tv, a)d(Sx_{2n}, Px_{2n}, a),$

$d(Sx_{2n}, Tv, a)d(Tv, Qv, a), d(Sx_{2n}, Tv, a)d(Sx_{2n}, Qv, a),$

$d(Sx_{2n}, Tv, a)d(Tv, Px_{2n}, a), d(Sx_{2n}, Px_{2n}, a)d(Tv, Qv, a),$

$d(Sx_{2n}, Px_{2n}, a)d(Sx_{2n}, Qv, a), d(Sx_{2n}, Px_{2n}, a)d(Tv, Px_{2n}, a),$

$d(Tv, Qv, a)d(Sx_{2n}, Qv, a), d(Sx_{2n}, Qv, a)d(Tv, Px_{2n}, a)).$

Taking $n \to \infty$, we get that

$d^2(u, Qv, a) \le \phi(0, 0, 0, 0, 0, 0, 0, 0, d^2(u, Qv, a), 0, 0,)$

$\le c(d^2(u, Qv, a)).$

Which implies that u = Qv = Tv = Pu. Taking pair (P, S) are weakly compatible, such that PSu = SPu = Pu = Su. Using again (b_5), we have

$d^2(Px_{2n}, Qu, a) \le \phi(d^2(Sx_{2n}, Tu, a), d(Sx_{2n}, Tu, a)d(Sx_{2n}, Px_{2n}, a),$

$d(Sx_{2n}, Tu, a)d(Tu, Qu, a), d(Sx_{2n}, Tu, a)d(Sx_{2n}, Qu, a),$

$d(Sx_{2n}, Tu, a)d(Tu, Px_{2n}, a), d(Sx_{2n}, Px_{2n}, a)d(Tu, Qu, a),$

$d(Sx_{2n}, Px_{2n}, a)d(Sx_{2n}, Qu, a), d(Sx_{2n}, Px_{2n}, a) d(Tu, Px_{2n}, a),$

$d(Sx_{2n}, Px_{2n}, a) d(Tu, Px_{2n}, a), d(Tu, Qu, a) d(Sx_{2n}, Qv, a),$

$d(Tu, Qu, a) d(Tu, Px_{2n}, a)).$

$n \to \infty$, we have

$d^2(u, Qu, a) \le \phi(d^2(u, Qu, a), 0, 0, d^2(u, Qu, a), d^2(u, Qu, a), 0, 0, 0, 0, 0, d^2(u, Qu, a)).$

$\le c(d^2(u, Qu, a)),$

Which means that u = Qu. Since $Q(Y) \subset S(Y)$, so there is w Y with u = sw. It follows from (b_5) that

$d^2(Px_{2n}, Qv, a) \le \phi(d^2(Sw, Tu, a), d(Sw, Tu, a)d(Sw, Pw, a),$

$$
\begin{aligned}
& d(Sw, Tu, a)d(Tu, Qu, a), d(Sw, Tu, a)d(Tu, Pw, a), \\
& \quad d(Sw, Pw, a)d(Tu, Qu, a), d(Sw, Pw, a)d(Sw, Qu, a), \\
& \quad d(Sw, Pw, a)d(Tu, Pw, a), d(Tu, Qu, a)d(Sw, Qu, a), \\
& \quad d(Tu, Qu, a)d(Tu, Pw, a), d(Sw, Qu, a)d(Tu, Pw, a)). \\
& = \phi(0, 0, 0, 0, 0, 0, 0, d^2(u, Pw, a), 0, 0, 0) \\
& \leq c(d^2(u, Pw, a)).
\end{aligned}
$$

This gives that u = Pw. Therefore, Sw = Pw. That is, u is a common fixed point ofsa P, Q, S and T.

Similarly , we can complete the proof when Q or S is continuous.

This completes the proof.

REFERENCE

1. Y. J. Cho, On existence of fixed points in 2-metric spaces, Pusan Kyo. Math. J. 1 (1985), 81-88.
2. Y. J. Cho, Fixed points for compatible mappings of type (A), Math. Japon. 38 (1993), 497-508.
3. Y. J. Cho M. S. Khan and S. L. Singh, Common fixed points of weakly commuting mappings, Univ. Novom Sadu, Zb. Rad. Prirod-Mat. Fak. Ser. Mat. 18 (1) (1988), 129-142.
4. K. J. Chung, Common fixed point theorems of two mappings, Comment. Math. Univ St. Pauli 24 (1975), 97-106.
5. A. Constantin, Common fixed points of weakly commuting mappings in 2-metric spaces, Math. Japon 36 (1991), 507-514.
6. C. Diminnie, S. Gähler and A. White Jr., 2-Inner product spaces, Demonst. Math. 6 (1973), 525-536.
7. B. Fisher and S. Sessa, Two common fixed point theorems for weakly commuting mappings, Periodica Math. Hung. 20 (1989), 207-218.
8. S. Gähler, 2-metrische Räume und ihre topoligische struktur, Math. Nachr. 26 (1963), 115-148.
9. M. Imdad, M. S. Khan and M. D. Khan, A common fixed point theorem in 2-metric spaces, Math. Japon 36(1991), 907-914.
10. G. Jungck, Commuting mappings and fixed points, Internat. J. Math. Math. Sci. 9 (1986), no. 4, 771-779.
11. G. Jungck, Compatible mapping and common fixed points, Internat. J. Math. Math. Sci. 9 (1986), 771-779.
12. G. Jungck, Common fixed points for commuting and compatible maps on compacta, Proc. Amer. Math. Soc. 103 (1988), 977-983.
13. G. Jungck, Compatible mappings and common fixed points, (2), Internat, J. Math. Math .Sci 11 (1988), 285-288.
14. H. Kaneko and S. Seesa, Fixed point theorems for compatible multivalued and single valued mappings, Internat. J. Math. Sci. 12 (1989), 257-262.
15. R. Kannan, Some results on fixed points, Bull. Cal. Math. Soc.60 (1968), 71-76.
16. M. S. Khan, On the convergence of sequence of fixed points in 2 -metric spaces, Indian J. Pure Appl. Math. 10 (1979), 1062-1067.
17. M. S. Khan and B. Fisher, Some fixed point theorems for commuting mappings, Math. Natur. 106 (1982), 323-326.
18. T. Kubiak, Common fixed points of pairwise commuting mappings, Math. Nachr. 118 (1984), 123-127.
19. Z. Liu, F. Zhang and J. Mao, Common fixed points for compatible mappings of type (A), Bull. Malaysian Math. Soc. 22 (1999), 67-86.
20. P. P. Murthy, S. S. Chang, Y. J. Cho and B. K. Sharma, Compatible mappings of type (A) and common fixed point theorems, Kyungpook Math. J. 32 (1992), 203-216.
21. R. P. Pant, Common fixed point of two pairs of commuting mappings, Indian J. Pure Appl. Math. 17 (1986), 187-192.
22. H. K. Pathak, Weak commuting mappings and fixed points, Indian J. Math. Anal. Appl. 188 (1986), 201-211.
23. H. K. Pathak, S. S. Chang and Y. J. Cho, Fixed point theorems for compatible mappings of type (P), Indian J. Math. 36(1994), 151-156.

24. H. K. Pathak and A. R. Maity, Fixed point theorems in 2-metric spaces, J. Indian Acad. Math. 12 (1990), 17-24.
25. B. E. Rhoades, Remarks on a paper of Chung, Comment Math. Univ. St. Pauli XXV (2) (1976), 115-116.
26. B. E. Rhoades and S. Sessa, Common fixed point theorems for three mappings under a weak commutativity condition, Indian J. Pure Appl. Math. 17 (1986), 47-57.
27. S. Sessa, On a weak commutativity condition of mappings in fixed point considerations, Publ. Inst. Math. (Beegrad) 32 (46) (1982) 149-153.
28. S. Seesa and B. Fisher, Some remarks on a fixed point theorem of T. Kubiak, Publ. math. Deb. 37 (1990), 41-45.
29. S. L. Singh and B. Ram, A note on convergence of sequences of mappings and their common fixed points in a 2-metric space, Math. Sem. Notes Kobe Univ. 9 (1981) 181-185.
30. S. L. Singh and B. Ram, Common fixed points of commuting mappings in 2-metric spaces, Math. Sem. Notes, Kobe Univ. 10 (1982), 177-208.
31. S. L. Singh and K. P. R. Rao, Coincidence and fixed points for four mappings, Indian J. Math. 31 (1989), 215-223.
32. S. L. Singh and B. M. L. Tivari, A note on a recent generalization of Jungck contraction principle, J. UPGC, Acad. Soc. 3 (1986), 13-18