American. Jr. of Mathematics and Sciences
Vol. 1, No.1 ,(January 2012)

www.journalshub.com

RIVLIN-ERICKSON ELASTICO-VISCOUS FLUID OF RELATED DENSITY

Lekh Raj
Research Scholar PhD(Mathematics)
Department of Mathematics,
CMJ University, SHILLONG.INDIA
E-mail id: lekhraj2007@rediffmail.com

Dr. Mohd. A.A.Ansari
Reader, Department of Mathematics,
Govt. P. G. College, Harakh, Barabanki (U.P.)

Abstract

A layer of compressible, rotating, elastico-viscous fluid of related density heated & soluted from below is considered in the presence of vertical magnetic field to include the effect of Hall currents. Dispersion relation governing the effect of visco-elasticity, salinity gradient, rotation, magnetic field and Hall currents is derived. For the case of stationary convection, the Rivlin-Erickson fluid behaves like an ordinary Newtonian fluid. The compressibility, stable solute gradient, rotation and magnetic field postpone the onset of thermo-solutal instability whereas Hall currents are found to hasten the onset of thermo-solutal instability in the absence of rotation. In the presence of rotation, Hall currents postpone/hasten the onset of instability depending upon the value of wave numbers. Again, the dispersion relation is analyzed numerically & the results depicted graphically.

Key Words:- Rivlin-Erickson elastico-viscous fluids, analogous solvent coefficient of expansion, resistivity, magnetic permeability, electron number density, transverse magnetic field etc.

Introduction

The case of over stability is discussed & sufficient conditions for non-existence of over stability are derived. For thermo-solutal convection, buoyancy forces can arise not only from density differences due to variation in temperature gradient, but also from those due to variation in solute concentration and this double diffusive phenomenon has been extensively studied in recent years due to its direct relevance in the field of chemical engineering, astrophysics, and oceanography. Veronis [20] studied the problem of thermo-haline convection in the layer of fluid heated from below and subjected to a stable salinity gradient. The physics is quite similar to Veronis thermo-haline configuration in the stellar case, in that helium acts like salt raising the density and in diffusing more slowly than heat. The heat and solute being two diffusing components, thermo-solutal (double-diffusive) convection is the general term dealing with such phenomenon. When the fluids are compressible, the equations governing the system become quite complicated. Spiegal and Veronis [18] have simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, if motions of infinitesimal amplitude are considered. Sharma and Gupta [16] have considered the effect of suspended particles and Hall currents on the stability of compressible fluids saturating a porous medium. Chandrasekhar [3] has given a detailed account of the theoretical and experimental results on the onset of thermal instability (B´enard convection) in an incompressible, viscous Newtonian fluid layer under varying assumptions of hydrodynamics and hydro-magnetics. In all these studies, fluid has been considered to be Newtonian.

One such class of elastico-viscous fluids is Rivlin-Erickson fluid. Joshi [9] has discussed the visco-elastic Rivlin-Erickson incompressible fluid of related density under time dependent pressure gradient. Gupta [6] studied the stability of stratified Rivlin- Erickson fluid of related density in the presence of variable magnetic field and uniform rotation in a porous medium and found the stabilizing role of magnetic field for a certain wave number range as in the case of Newtonian fluids. The study of viscoelastic fluids has become of increasing importance due to their application in petroleum industry, food and chapter industry, and similar activities. **Mathematical and graphically formulation :-** Rivlin-Erickson fluids of related densities are characterized by the constitutive equations [13]

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2 + \mu_4 A_2^2 +$$

$$\mu_5 (A_1 A_2 + A_2 A_1) + \mu_6 (A_1^2 A_2 + A_2 A_1^2) +$$

$$\mu_7 (A_1 A_2^2 + A_2^2 A_1) + \mu_6 (A_1^2 A_2^2 + A_2^2 A_1^2)$$

(1)

where *S* is Cauchy stress tensor, *p* is an arbitrary hydrostatic pressure, *I* is the unit tensor and $\mu_i{}'s$ are polynomial functions of the traces of the various tensors occurring in the representation. A_1, A_2 are Rivlin-Erickson tensors and denote respectively the rate of strain and acceleration which are defined as

$$A_1 = (grad\ q) + (grad\ q)^T \qquad (2)$$

$$A_2 = (grad\ a) + (grad\ a)^T + 2(grad\ q)(grad\ q)^T \qquad (3)$$

In the above equations a is the acceleration in substantial formulation, q is velocity vector. Neglecting the squares and products of A_2, we get Rivlin- Erickson fluid as

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2 \qquad (4)$$

where μ_1, μ_2, μ_3 are measurable material constants. They denote respectively the coefficient of ordinary viscosity, the coefficient of visco-elasticity, and the coefficient of cross-viscosity and are in general functions of temperature and material properties. The visco-elastic fluid when modelled by the Rivlin-Erickson constitutive equations are termed second-order fluids. Second-order fluids are dilute polymeric solutions(e.g. poly-iso-butylene, methyl methacrylate in nbutylacetate, polyethylene oxide in water etc.). Rathna [12] has shown that fluid is visco-elastic if μ_3 is zero and non-Newtonian fluid with cross viscosity if $\mu_2 = 0$. Heuristically, this approximation should hold in the case of second-order fluids.

Recently, Halder [7] investigated the flow of blood through a constricted artery in the presence of an external transverse magnetic field using Adomian's decomposition method. The expressions for two term approximation to the solution of stream function, axial velocity component and wall shear stress are obtained in this analysis. Sharma and Kumar [15] have studied the effect of rotation on thermal instability in Rivlin-Erickson elastico-viscous fluids. Recently, Sunil et. al. [19] have studied the effect of Hall currents on thermo-solutal instability of compressible Rivlin-Erickson fluids. Keeping in mind the conflicting tendencies of magnetic field and rotation while acting together and the growing importance of non-Newtonian fluids in modern technology, industry, chemical technology and dynamics of geophysical fluids; we are motivated to study the thermo-solutal instability of a compressible Rivlin-

Erickson fluid in the presence of rotation and Hall currents. This problem to the best of our knowledge, has not been investigated yet.

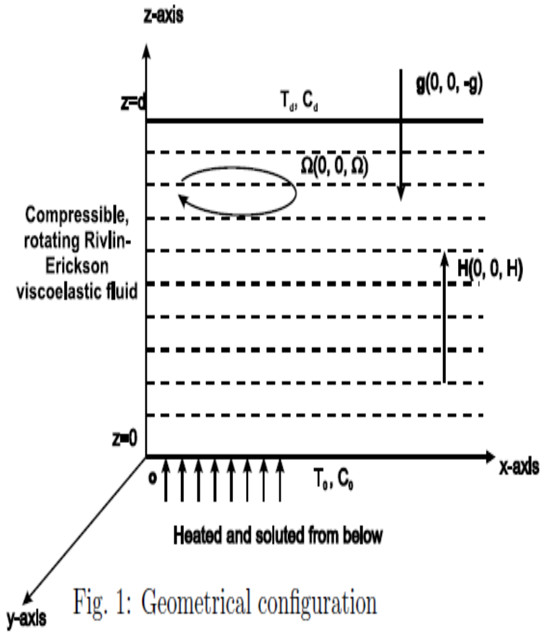

Fig. 1: Geometrical configuration

Formulation of the problem and perturbation equations: We have considered an infinite, horizontal, compressible electrically conducting Rivlin-Erickson fluid layer of thickness *d* which is heated and soluted from below(at *z* = 0) so that temperature and concentration at bottom is T_0 and C_0 and at the upper surface (*z* = *d*) is T_d and C_d respectively. A uniform temperature gradient and concentration gradient, $\beta(=|dT/dz|)$ and $\beta'(=|dC/dz|)$ maintained. The elastico-viscous fluid is acted on by gravity force $g(0,0,-g)$, a uniform vertical rotation $\Omega(0,0,\Omega)$ and a uniform vertical magnetic field $H(0,0,H)$. In the present chapter we are considering all the assumptions that lead to Oberbeck-Boussinesq system and assume that material constant μ_3 is zero (neglecting the cross viscosity effect) following Dunn and Rajagopal [4] in considering μ_2 > 0 to study the visco-elastic effect on the onset of convection.

Let $p, \rho, T, C, \alpha, \alpha', g, \eta, \mu_e, N, e, \upsilon, \upsilon', k, k'$ denote, respectively, the pressure, density, temperature, concentration, thermal coefficient of expansion, analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron, kinematic viscosity, kinematic visco-elasticity, thermal diffusivity, solute diffusivity, and fluid velocity. The equations expressing conservation of momentum, mass, temperature, solute concentration, and equation of state of a Rivlin-Erickson fluid (Chandrasekhar [3]; Rivlin and Erickson [13]; Joseph [8]) are

$$\left[\frac{\partial q}{\partial t}+(q.\nabla)q\right]=-\left(\frac{1}{\rho_m}\right)\nabla p+g\left(1+\frac{\delta\rho}{\rho_m}\right)$$

$$+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 q+\frac{\mu_e}{4\pi\rho_m}(\nabla\times H)\times H+2(q\times\Omega) \quad (5)$$

$$\nabla.q=0 \quad (6)$$

$$\frac{\partial T}{\partial t} + (\nabla.q)\,T = k\nabla^2 T \qquad (7)$$

$$\frac{\partial C}{\partial t} + (\nabla.q) = k'\nabla^2 C \qquad (8)$$

$$\rho = \rho_m[1-\alpha(T-T_0)+\alpha'(C-C_0)] \qquad (9)$$

In the present model, we have ignored the non-Newtonian effects of second order fluids on heat transportation in comparison to other terms in heat equation and assume that visco-elastic effects influence the heat transport only through velocity. From Maxwell's equations, we have

$$\frac{\partial H}{\partial t} = (H.\nabla).q + \eta\nabla^2 H - \frac{c}{4\pi Ne}\nabla\times[(\nabla\times H)\times H] \qquad (10)$$

$$\nabla.H = 0 \qquad (11)$$

Where $\frac{d}{dt} = \frac{\partial}{\partial t} + q.\nabla$ stands for convective derivative. The state variables pressure, density, and temperature, are expressed in the form (Spiegal and Veronis [18])

$$f(x,y,z,t) = f_m + f_0(z) + f'(x,y,z,t) \qquad (12)$$

f_m stands for constant space distribution of f, f_0 is the variation in the absence of motion and $f'(x,y,z,t)$ is the fluctuation resulting from motion. For initial state, we have

$$p = p(z), \rho = \rho(z), T = T(z), C = C(z),$$

$q = (0,0,0)$ and $H = (0,0,H)$

Where $p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz$

$$\rho(z) = \rho_m[1-\alpha_m(T-T_0)+\alpha_m'(C-C_0)$$
$$+K_m(p-p_m)]$$

$$T(z) = -\beta z + T_0, T(z) = -\beta' z + C_0$$

$$\alpha_m = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m (=\alpha,) \text{ say}$$

$$\alpha_m' = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial C}\right)_m (=\alpha',) \text{ say}$$

$$K_m = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m (=\alpha,) \qquad (13)$$

Here p_m and ρ_m stand for a constant space distribution of p and ρ. Linearized stability theory and normal mode analysis method is used to study infinitesimal perturbations and depth of fluid layer is assumed to be much less than the scale height as defined by Spiegal and Veronis [18].Let us consider a small perturbation on steady state solution and let $\delta p, \delta\rho, \theta, h = (h_x, h_y, h_z)$ and $\gamma, h = (h_x, h_y, h_z)$ denote, the perturbations in pressure, density, temperature, solute concentration, magnetic field, and velocity respectively. The change in density $\delta\rho$ is given by

$$\delta\rho = -\rho_m(\alpha\theta - \alpha'\gamma) \qquad (14)$$

Then the linearized hydromagnetic perturbation equations are

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho_m}(\nabla\delta p) - g(\alpha\theta - \alpha'\gamma) +$$

$$\left(\nu + \nu'\frac{\partial}{\partial t}\right)\nabla^2 q + \frac{\mu_e}{4\pi\rho_m}(\nabla\times h)\times H + 2(q\times\Omega)$$

$$(15)$$

$$\nabla.q = 0 \qquad (16)$$

$$\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)w + k\nabla^2\theta \qquad (17)$$

$$\frac{\partial\gamma}{\partial t} = \beta' w + k'\nabla^2\gamma \qquad (18)$$

$$\nabla.h = 0 \qquad (19)$$

$$\frac{\partial h}{\partial t} = (H.\nabla)q + \eta\nabla^2 H - \frac{c}{4\pi Ne}\nabla\times[(\nabla\times h)\times H] \qquad (20)$$

Dispersion relation: In normal mode analysis method, let us assume that perturbation quantities are of the form

$$[w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), X(z)] \times \exp(ik_x x + ik_y y + nt), \qquad (21)$$

where k_x, k_y are the wavenumbers along x,y directions and resultant wave number is given by $k = \sqrt{k_x^2 + k_y^2}$ and n is the growth rate. $\zeta = \frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density. Using expression (21), Eqs. (15) − (20), take the forms

$$[(1+\sigma F)(D^2-a^2)-\sigma](D^2-a^2)W-\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma)$$
$$+\frac{\mu_e Hd}{4\pi\rho_m\nu}(D^2-a^2)DK-\frac{2\Omega d^3}{\nu}DZ=0, \qquad (22)$$

$$[(1+\sigma F)(D^2-a^2)-\sigma]Z+\frac{\mu_e Hd}{4\pi\rho_m\nu}DX+\frac{2\Omega d}{\nu}DZ=0, \qquad (23)$$

$$(D^2-a^2-p_2\sigma)K+\left(\frac{Hd}{\eta}\right)DW-\frac{cHd}{4\pi Ne\eta}DX=0, \qquad (24)$$

$$(D^2-a^2-p_2\sigma)X+\left(\frac{Hd}{\eta}\right)DZ+\frac{cH}{4\pi Ne\eta d}(D^2-a^2)DK=0, \qquad (25)$$

$$(D^2-a^2-p_1\sigma)\Theta+\frac{gd^2}{\kappa C_p}(G-1)W=0, \qquad (26)$$

$$(D^2-a^2-q\sigma)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W, \qquad (27)$$

where we have non-dimesionalized the various parameters as follows:

$$a=kd,\ \sigma=\frac{nd^2}{\nu},\ p_1=\frac{\nu}{\kappa},\ p_2=\frac{\nu}{\eta},\ q=\frac{\nu}{\kappa'},\ F=\frac{\nu'}{d^2},$$
$$G=\left(\frac{C_p}{g}\right)\beta,\ x^*=\frac{x}{d},\ y^*=\frac{y}{d},\ z^*=\frac{z}{d}\text{ and } \mathrm{D}=\frac{\mathrm{d}}{\mathrm{dz}^*}.$$

We consider the case of two free boundaries which are perfect conductors of both heat and solute concentration. The case of two free boundaries is of little physical interest but is mathematically very important as it enables us to get analytical solutions and draw some qualitative conclusions. For the case of free boundaries the boundary conditions are (Chandrasekhar[3])

$W=D^2w=0,\ Dz=0,\ \Theta=0,\ \Gamma=0$ at z = 0 and 1, K = 0 on perfectly conduction boundaries. (28)

and h_x, h_y, h_z are continuous. Using these boundary conditions, it can be shown that all the even order derivatives of *W* must vanish for *z* = 0 and 1. Therefore proper solution of *W* characterizing the lowest mode is $W=w_0\sin\pi z$ (29)

where w_0 is a constant. After eliminating Θ, X, Z, Γ and K between Eqs. (22) − (27), we obtain

$$R_1 = \left(\frac{G}{G-1}\right)\left\langle \left(\frac{1+x}{x}\right)[(1+x)(1+i\sigma_1\pi^2F)+i\sigma_1](1+x+i\sigma_1p_1) \right.$$
$$+ S_1\frac{(1+x+i\sigma_1p_1)}{(i+x+i\sigma_1q)} + \frac{(1+x+i\sigma_1p_1)}{x}\left[Q_1(1+x)\right.$$
$$\times\left\{[(1+x)(1+i\sigma_1\pi^2F)+i\sigma_1](1+x+i\sigma_1p_2)+Q_1+2\sqrt{T_1M}\right\}$$
$$\left.+T_1\left\{(1+x+i\sigma_1p_2)^2+M(1+x)\right\}\right]$$
$$\left\{(1+x+i\sigma_1p_2)^2[(1+x)(1+i\sigma_1\pi^2F)+i\sigma_1]+Q_1(1+x+i\sigma_1p_2)\right.$$
$$\left.\left.+M(1+x)[(1+x)(1+i\sigma_1\pi^2F)+i\sigma_1]\right\}^{-1}\right\rangle, \quad (30)$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4},\ S_1 = \frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4},\ Q_1 = \frac{\mu_e H^2 d^2}{4\pi\rho m\nu\eta\pi^2},\ M = \left(\frac{cH}{4\pi Ne\eta}\right)^2,\ T_1 = \frac{4\Omega^2 d^4}{\nu^2\pi^4}, \quad x = \frac{a^2}{\pi^2},\ \text{and}\ i\sigma_1 = \frac{\sigma}{\pi^2}.$$

Equation (30) is the required dispersion relation including the effects of rotation, Hall currents, compressibility and solute gradient on the thermosolutal instability of a Rivlin-Erickson fluid. This equation reduces to the dispersion relation obtained by Sunil et.al. [19], in the absence of rotation.

Conclusion:- We have investigated the effects of rotation, magnetic field and Hall currents on the stability of a compressible Rivlin-Erickson elastico-viscous fluid of related density heated and soluted from below. We derived the dispersion relation including the effects of rotation, Hall currents, compressibility and solute gradient on the thermosolutal instability of a Rivlin-Erickson fluid. The principal conclusions from the analysis of this chapter are as follows:

(i) These analytical results are well supported numerically/graphically as can be seen from Fig. The reasons for stabilizing effects of magnetic field and rotation are accounted by Chandrasekhar [3] and for stable solute gradient by Veronis [20]. These are valid for second-order fluids as well. Gupta [5] observed the destabilizing effect of Hall currents for the case of Newtonian fluids. While studying the effect of one parameter graphically, on the stability of the system, values of other parameters are kept at the lowest level in the chosen range. This ensures a minimum possible interaction of the other parameters which may otherwise influence the stability of the system.

References

1.Alterman Z (1961). Effect of surface tension to the Kelvin-Helmholtz instability of two rotating fluids. Nat. Acad. Sci. (U.S.A.) 47(2), pp. 224 -227.

2. Bellman R and Pennigton RH (1954). Effect of surface tension and viscosity on Taylor instability. Quart. Appl. Math. 12, pp.151–162.

3. Chandrasekhar S (1961). Hydrodynamic and Hydromagnetic Stability. Clarendon, Oxford.
Chouke RL, van Meurs P and van der Poel C (1959). The instability of slow immiscible viscous liquid-liquid-displacements in permeable media. Trans. AIME 216, pp. 188-194.

4. Ericksen JL (1953). Characterstic surfaces of equations of motion of non-Newtonian fluids. ZAMP 4, pp. 260-267.

5. Fredricksen AG (1964). Principles and applications of Rheology. Prentice-Hall Inc., New Jersey.

6. Gerwin RA (1968). Stability of the interface between two fluids in relative motion. Review of Modern Physics 40(3), pp. 652-658.

7. Goren SL and Gottlieb M (1982). Surface –tension-driven breakup of viscoelastic liquid threads. J. Fluid Mech. 120, pp. 245-266.

8. Helmholtz H (1868). Ueber discontinuirlicke Flussigkeitsbewgungen. Phil. Mag. Ser. 4(36), pp. 337-346.
Joseph DD (1976). Stability of fluid motion II. Springer–Verlag, New York.

9. Kelvin L (1910). Mathematical and Physical Chapters, IV, Hydrodynamics and General Dynamics. Cambridge, England, pp. 67-85.

10. Kent A (1966). Instability of laminar flow of a magnetofluid. Phys. Fluids 9, pp. 1286- 1289.

11. McDonnel JAM (1978). Cosmic Dust. John Wiley and Sons, Toronto, p.330.

12. Reid WH (1961). The effects of surface tension and viscosity on the stability of two superposed fluids, Proc. Camb. Phil. Soc. 57, pp. 415-425.

13. Rivlin RS (1948). The hydrodynamics of non-Newtonian fluids, I. Proc.Roy.soc. (London)A193, pp.260 -281.

14. Sharma RC and Kango SK (1999). Stability of two superposed Walters' (model B□□) viscoelastic fluids in the presence of suspended particles and variable magnetic field in porous medium. Appl.Mech. Engng. 4(2), pp. 255-269.

15. Sharma V and Kumar S (2001). Rayleigh -Taylor instability of stratified Walters'(model B□□) fluid in the presence of a variable horizontal magnetic field and suspended particles. Journal of Indian Math. Soc. 68, pp.209-219.

16. Sharma V and Kumar S (2000). Magnetogravitational instability of a thermally conducting rotating Walters' (model B□□) fluid with Hall current. Proc. Nat. Acad. Sci. India 70(A) I, pp. 87-97.
17. Sharma RC and Spanos TJT (1982). The instability of streaming fluids in a porous medium. Canadian J. Phys. 60, pp. 1391-1395.