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HAUSDORFF EXTENSIONS OF SPACES IN INTUITIONISTIC FUZZY $S*$ CENTRED SYSTEMS

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Abstract

In this paper, *the concepts of* ***maximal intuitionistic fuzzy $S*$ centred systems and Hausdorff extensions of spaces in intuitionistic fuzzy $S*$ centred system are studied as in [9] and their properties are discussed.***

Key words

Intuitionistic fuzzy $S*$ structure space $(S, S*)$, intuitionistic fuzzy $S*$ centred system, a maximal intuitionistic fuzzy $S*$ centred system, intuitionistic fuzzy $S*H$ -closed, intuitionistic fuzzy $S*$compact, intuitionistic fuzzy S^* completely regular space, intuitionistic fuzzy S^* maximal compact extension $\beta(S)$.

1.Introduction

After the introduction of the concept of fuzzy sets by Zadeh [13], several researches were conducted on the generalizations of the notion of fuzzy set. The concept of "Intuitionistic fuzzy sets" was first published by Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2-4]. Later this concept was generalized to "Intuitionistic L-fuzzy sets" by Atanassov and stoeva [5]. An introduction to intuitionistic fuzzy topological space was introduced by Dogan Coker [6]. Fuzzy continuity in intuitionistic fuzzy topological spaces was introduced by Gurcay. H, Coker, D., and Haydar, Es. A., [8]. The method of centered system in the theory of topological spaces was introduced by Iliadis. S, Fomin.S [9], in order to study, as far as possible from one point of view, a number of problems in the theory of extension of topological spaces, like compactness and H-closure, as well as a newer range of ideas concerned principally with the work of Gleason [7] and Ponomarev [10]. The concept of intuitionistic fuzzy compactness was found in [6] and intuitionistic fuzzy Hausdorff space was found in [6]. Uma, Roja and Balasubramanian [12] extended the method of centred systems to fuzzy topological spaces. In this chapter, the concepts of maximal intuitionistic fuzzy $S*$ centred systems, intuitionistic fuzzy $S*$ Hausdorff extensions of spaces in intuitionistic fuzzy $S*$ centred system, the realization of an arbitrary intuitionistic fuzzy $S*$extension of a space of intuitionistic fuzzy $S*$ centered systems, intuitionistic fuzzy

S^* Hausdorff extensions of types σ and τ, maximality of intuitionistic fuzzy S^* spaces $\sigma(S)$ and $\tau(S)$ and classes of intuitionistic fuzzy S^* Hausdorff extensions of *S* are studied.

2.Preliminaries

Definition 2.1[3] Let X be a non empty fixed set. An intuitionistic fuzzy set (*IFS* for short) A is an object having the form $A=\left\{\left\langle x,\mu_A(x),\gamma_A(x)\right\rangle : x\in X\right\}$ where the functions $\mu_A : X\to I$ and $\gamma_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$.

Remark 2.1[6] For the sake of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$.

Definition 2.2[6] Let X be a non empty set and let $\{A_i : i\in J\}$ be an arbitrary family of *IFSs* in X. Then (a) $\bigcap A_i=\left\{\left\langle x,\wedge\mu_{A_i}(x),\vee\gamma_{A_i}(x)\right\rangle : x\in X\right\}$;

(b) $\bigcup A_i=\left\{\left\langle x,\vee\mu_{A_i}(x),\wedge\gamma_{A_i}(x)\right\rangle : x\in X\right\}$.

Definition 2.3[6] Let X be a non empty fixed set. Then, $0_\sim=\left\{\left\langle x,0,1\right\rangle : x\in X\right\}$ and $1_\sim=\left\{\left\langle x,1,0\right\rangle : x\in X\right\}$.

Definition 2.4[6] Let X and Y be two non empty fixed sets and $f: X\to Y$ be a function. Then

(a) If $B=\left\{\left\langle y,\mu_B(y),\gamma_B(y)\right\rangle : y\in Y\right\}$ is an *IFS* in *Y*, then the pre image of B under f, denoted by $f^{-1}(B)$, is the *IFS* in X defined by $f^{-1}(B)=\left\{\left\langle x,f^{-1}(\mu_B)(x),f^{-1}(\gamma_B)(x)\right\rangle : x\in X\right\}$.

(b) If $A=\left\{\left\langle x,\lambda_A(x),\nu_A(x)\right\rangle : x\in X\right\}$ is an *IFS* in X, then the image of A under f, denoted by $f(A)$, is the *IFS* in *Y* defined by

$f(A)=\left\{\left\langle y,f(\lambda_A)(y),(1-f(1-\nu_A))(y)\right\rangle : y\in Y\right\}$ where,

$$f(\lambda_A)(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)}\lambda_A(x) & \text{if } f^{-1}(y)\neq 0\\ 0, & \text{otherwise,}\end{cases}$$

$$(1-f(1-\nu_A))(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\nu_A(x) & \text{if } f^{-1}(y)\neq 0\\ 1, & \text{otherwise.}\end{cases}$$

for the *IFS* $A=\left\{\left\langle x,\mu_A(x),\gamma_A(x)\right\rangle : x\in X\right\}$.

Definition 2.5[6] Let X be a non empty set. An intuitionistic fuzzy topology (*IFT* for short) on a non empty set X is a family τ of intuitionistic fuzzy sets (*IFSs* for short) in X satisfying the following axioms: (T_1) $0_\sim,1_\sim\in\tau$, (T_2) $G_1\cap G_2\in\tau$ for any $G_1,G_2\in\tau$, (T_3) $\bigcup G_i\in\tau$ for any arbitrary family $\{G_i : i\in J\}\subseteq\tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (*IFTS* for short) and any *IFS* in τ is known as an intuitionistic fuzzy open set (*IFOS* for short) in X .

Definition 2.6[6] Let X be a non empty set. The complement $\overline{A}$ of an *IFOS* A in an *IFTS* (X,τ) is called an intuitionistic fuzzy closed set (*IFCS* for short) in X .

Definition 2.7[6] Let (X,τ) be an *IFTS* and $A=\langle x,\mu_A,\gamma_A\rangle$ be an *IFS* in X . Then the fuzzy interior and fuzzy closure of A are defined by

$cl(A)=\bigcap\{K:K$ is an *IFCS* in X and $A\subseteq K\}$,

$\text{int}(A)=\bigcup\{G:G$ is an *IFOS* in X and $G\subseteq A\}$.

Remark 2.2[6] Let (X,τ) be an *IFTS*. $cl(A)$ is an *IFCS* and $\text{int}(A)$ is an *IFOS* in X , and (a) A is an *IFCS* in X iff $cl(A)=A$; (b) A is an *IFOS* in X iff $\text{int}(A)=A$.

Proposition 2.1[6] Let (X,τ) be an *IFTS*. For any *IFS* A in (X,τ), we have

(a) $cl(\overline{A})=\overline{\text{int}(A)},$ (b) $\text{int}(\overline{A})=\overline{cl(A)}.$

Definition 2.8[6] Let (X,τ) and (Y,φ) be two *IFTSs* and let $f:X\to Y$ be a function. Then f is said to be fuzzy continuous iff the pre image of each *IFS* in ϕ is an *IFS* in τ.

Definition 2.9[6] Let (X,τ) and (Y,φ) be two *IFTSs* and let $f:X\to Y$ be a function. Then f is said to be fuzzy open(resp.closed) iff the image of each *IFS* in τ (resp.(1- τ)) is an *IFS* in ϕ (resp.(1-ϕ)).

Definition 2.10[11] A fuzzy topological space (X,T) is called fuzzy Hausdorff space iff for any two distinct fuzzy points $p,q\in X,$ there exists $U,V\in T$ such that $U\bigcap V=0$ with $p\in U$ and $q\in V.$

Definition 2.11[12] Let R be a fuzzy Hausdorff space. A system $p=\{U_\alpha\}$ of fuzzy open sets of R is called fuzzy centred if any finite collection of the sets of the system has a non empty intersection. The system p is called a maximal fuzzy centred system or a fuzzy end if it cannot be included in any larger fuzzy centred system of fuzzy open sets.

Definition 2.12[9] A Hausdorff space $\delta(R)$ is called an extension of a Hausdorff space R if R is contained in $\delta(R)$ as an everywhere dense subset. R is called H - closed if every extension coincides with R itself. An extension $\delta(R)$ is called H -closed if $\delta(R)$ is H - closed and compact if $\delta(R)$ is compact.

3.1 INTUITIONISTIC FUZZY $S*$ HAUSDORFF EXTENSIONS OF SPACES

In this section, three criteria for intuitionistic fuzzy *H*-closure are studied and interesting properties and characterizations are established.

Definition 3.1 Let *X* be a non empty set. Let S be a collection of all intuitionistic fuzzy sets of *X*. An intuitionistic fuzzy $S*$structure on S is a collection $S*$ of subsets of S having the following properties

(a) ϕ and S are in $S*$.

(b) The union of the elements of any sub collection of $S*$ is in $S*$.

(c) The intersection of the elements of any finite sub collection of $S*$ is in $S*$.

The collection S together with the structure S^* is called intuitionistic fuzzy S^* structure space. A subset U of S is an intuitionistic fuzzy S^* open set if $U \in S^*.$ Its complement is said to be an intuitionistic fuzzy S^* closed set.

Definition 3.2 Let A be a member of S. An intuitionistic fuzzy S^* open set U in (S, S^*) is said to be an intuitionistic fuzzy S^* neighbourhood of A if $A \in G \subset U$ for some intuitionistic fuzzy S^* open set G in (S, S^*).

Definition 3.3 Let (S, S^*) be an intuitionistic fuzzy S^* structure space and $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy S^* closure and intuitionistic fuzzy S^* interior of A are respectively defined and denoted by

$$IF\, S^* cl(A) = \bigcap \{ K : K \text{ is an intuitionistic fuzzy } S^* \text{closed sets in } S \text{ and } A \subseteq K \},$$

$$IF\, S^* \operatorname{int}(A) = \bigcup \{ G : G \text{ is an intuitionistic fuzzy } S^* \text{open sets in } S \text{ and } G \subseteq A \}.$$

Definition 3.4 The ordered pair (S, S^*) is called an intuitionistic fuzzy S^* Hausdorff space if for each pair A_1, A_2 of disjoint members of S, there exist disjoint intuitionistic fuzzy S^* open sets U_1 and U_2 such that $A_1 \subset U_1$ and $A_2 \subset U_2$.

Definition 3.5 Let (S_1, S_1^*) and (S_2, S_2^*) be any two intuitionistic fuzzy S^* structure spaces and let $f : (S_1, S_1^*) \to (S_2, S_2^*)$ be a function. Then f is said to be an intuitionistic fuzzy S^* continuous iff the pre image of each intuitionistic fuzzy S_2^* open set in (S_2, S_2^*) is an intuitionistic fuzzy S_1^* open set in (S_1, S_1^*).

Definition 3.6 Let (S_1, S_1^*) and (S_2, S_2^*) be two intuitionistic fuzzy S^* structure spaces and let $f : (S_1, S_1^*) \to (S_2, S_2^*)$ be a bijection function. If both the function f and the inverse function $f^{-1} : (S_2, S_2^*) \to (S_1, S_1^*)$ are intuitionistic fuzzy S^* continuous, then f is called a intuitionistic fuzzy S^* homeomorphism.

Definition 3.7 Let f be a function from an intuitionistic fuzzy S^* structure space (S_1, S_1^*) into an intuitionistic fuzzy S^* structure space (S_2, S_2^*) with $f(A_1) = A_2$ where $A_1 \in (S_1, S_1^*)$ and $A_2 \in (S_2, S_2^*)$. Then f is called an intuitionistic fuzzy S^* θ-continuous at A_1 if for every neighbourhood O_{A_2} of A_2 there exists a neighbourhood O_{A_1} of A_1 such that $f(IFS^*cl(O_{A_1}) \subset IFS^*cl(O_{A_2})$. The function is called intuitionistic fuzzy S^* θ-continuous if it is intuitionistic fuzzy S^* θ-continuous at every member of S_1. A function is called an intuitionistic fuzzy S^* θ-homeomorphism if it is intuitionistic fuzzy S^* one to one and intuitionistic fuzzy S^* θ-continuous in both directions.

Definition 3.8 Let (S, S^*) be an intuitionistic fuzzy S^* Hausdorff space. A system $p = \{U_\alpha : \alpha = 1, 2, 3, \ldots .n\}$ of intuitionistic fuzzy S^* open sets is called an intuitionistic fuzzy S^* centred system if any finite collection of the sets of the system has a non empty intersection. The intuitionistic fuzzy S^* centred system p is called a maximal intuitionistic fuzzy $S*$ centred system or an intuitionistic fuzzy $S*$ end if it cannot be included in any larger intuitionistic fuzzy $S*$ centred system of intuitionistic fuzzy $S*$ open sets.

Note 3.1 Throughout this chapter let $U_\alpha : \alpha = 1, 2, 3, \ldots n$ be an intuitionistic fuzzy $S*$ open set in $(S, S*)$.

Proposition 3.1 Let $(S,S*)$ be an intuitionistic fuzzy $S*$ Hausdorff space and $p = \{U_\alpha\}$ is an intuitionistic fuzzy $S*$ centred system in $(S, S*)$. Then the following properties hold

a. If $U_i \in p \quad (i = 1, 2, \ldots n)$, then $\bigcap_{i=1}^{n} U_i \in p.$

b. If $\phi \neq U \subseteq H,\ U \in p$ and H is intuitionistic fuzzy $S*$ open set, then $H \in p$.

c. If H is an intuitionistic fuzzy $S*$ open set, then $H \notin p$ iff there exists $U \in p$ such that $U \bigcap H = \phi.$

d. If $U_1 \bigcup U_2 = U_3 \in p$, U_1 and U_2 are intuitionistic fuzzy $S*$ open sets and $U_1 \bigcap U_2 = \phi,$ then either $U_1 \in p$ or $U_2 \in p.$

e. If $IFS*cl(U) = S,$ then $\phi \neq U \in p$ for any intuitionistic fuzzy $S*$ end p.

Definition 3.9 Let $\theta(S)$ denote the collection of all intuitionistic fuzzy $S*$ ends belonging to S. An intuitionistic fuzzy $S*$ topology is introduced into $\theta(S)$ in the following way. Let O_U be the set of all intuitionistic fuzzy $S*$ ends that contains U as an element, where U is an intuitionistic fuzzy $S*$ open set of S. Therefore O_U is an intuitionistic fuzzy $S*$ neighbourhood of each intuitionistic fuzzy $S*$ end contained in O_U .

Definition 3.10 A subset *A* of an intuitionistic fuzzy $S*$ structure space $(S,S*)$ is said to be an everywhere intuitionistic fuzzy $S*$ dense subset in $(S,S*)$ if $IFS^*cl(A) = S.$

A subset of an intuitionistic fuzzy $S*$ structure space $(S,S*)$ is said to be an nowhere intuitionistic fuzzy $S*$ dense subset in $(S,S*)$ if $X \setminus \overline{A}$ is everywhere intuitionistic fuzzy $S*$ dense subset.

Definition 3.11 Let $(S,S*)$ be an intuitionistic fuzzy $S*$ structure space and *Y* be an intuitionistic fuzzy $S*$ open set in $(S,S*)$. Then the intuitionistic fuzzy $S*$ relative topology $T_Y = \{G \bigcap Y : G \in S*\}$ is called the intuitionistic fuzzy $S*$ relative (or induced or subspace) topology on *Y*. The ordered pair (Y, T_Y) is called an intuitionistic fuzzy $S*$ subspace of the intuitionistic fuzzy $S*$ space $(S, S*)$.

Definition 3.12 Let $(S,S*)$ be an intuitionistic fuzzy $S*$ structure space. Then

(a) If a family $\{U_\alpha : i \in \Lambda\}$ of intuitionistic fuzzy $S*$ open sets in $(S,S*)$ satisfies the condition $S = \bigcup \{U_\alpha : i \in \Lambda\},$ then it is called an intuitionistic fuzzy $S*$ open cover of *S*. A finite subfamily of the intuitionistic fuzzy $S*$ open cover $\{U_\alpha : i \in \Lambda\}$ of *S*, which is also an intuitionistic fuzzy $S*$ open cover of *S*, is called a finite subcover of $\{U_\alpha : i \in \Lambda\}$.

(b) An intuitionistic fuzzy S^* structure space (S,S^*) is called intuitionistic fuzzy S^* compact iff every intuitionistic fuzzy S^* open cover of S has a finite subcover.

Definition 3.13 An intuitionistic fuzzy S^* Hausdorff space $\delta(S)$ is called an extension of an intuitionistic fuzzy S^* Hausdorff space S if S is contained in $\delta(S)$ as an everywhere intuitionistic fuzzy S^* dense subset. S is called an intuitionistic fuzzy S^*H-closed if every extension coincides with S itself. An extension $\delta(S)$ is called an intuitionistic fuzzy S^*H-closed if $\delta(S)$ is an intuitionistic fuzzy S^*H-closed, and intuitionistic fuzzy S^* compact if $\delta(S)$ is an intuitionistic fuzzy S^* compact.

Definition 3.14 Let (S,S^*) be an intuitionistic fuzzy S^* structure space. A system $\mathcal{B}$ of intuitionistic fuzzy S^* open sets of an intuitionistic fuzzy S^* structure space S is called an intuitionistic fuzzy S^* base(or basis) for (S,S^*) if each member of (S,S^*) is a union of members of $\mathcal{B}$**.** A member of $\mathcal{B}$ is called an intuitionistic fuzzy S^* basic open set.

Definition 3.15 Let (S,S^*) be an intuitionistic fuzzy S^* structure space. A system of intuitionistic fuzzy S^* open sets of an intuitionistic fuzzy S^* structure space S is called an intuitionistic fuzzy S^* sub-base if it together with all possible finite intersections of members of the system forms a base of S.

Lemma 3.1 An intuitionistic fuzzy S^* structure space S is intuitionistic fuzzy S^*H-closed if and only if any intuitionistic fuzzy S^* centered system $\{U_\alpha\}$ of intuitionistic fuzzy S^* open sets of S satisfies the condition

$$\bigcap_{\alpha} IFS^*cl(U_\alpha) \neq \phi$$

Lemma 3.2 An intuitionistic fuzzy S^* structure space S is intuitionistic fuzzy S^*H-closed if and only if any maximal intuitionistic fuzzy S^* centered system $\{U_\alpha\}$ of intuitionistic fuzzy S^* open sets of S contains all the intuitionistic fuzzy S^* neighbourhoods of some member.

The proof of this lemma follows easily from Lemma 3.1.

Lemma 3.3 The intuitionistic fuzzy S^* structure space S is intuitionistic fuzzy S^*H-closed if and only if from any intuitionistic fuzzy S^* cover $\{U_\alpha\}$ of S a finite subsystem U_i (i = 1,2,3,….,n) may be chosen such that

$$\bigcup_{i=1}^{n} IFS^*cl(U_i) = S.$$

The proof of this lemma follows easily from Lemma 3.1.

4.THE REALIZATION OF AN ARBITRARY INTUITIONISTIC FUZZY S^* EXTENSION OF A SPACE OF INTUITIONISTIC FUZZY S^* CENTERED SYSTEMS.

In this section, intuitionistic fuzzy S^* extensions of types σ and τ, the intuitionistic fuzzy S^* structure space $\sigma(S)$, intuitionistic fuzzy S^* structure space $\tau(S)$, maximality of the intuitionistic fuzzy S^* structure spaces $\sigma(S)$ and $\tau(S)$ and classes of intuitionistic fuzzy S^* Hausdorff extensions of S are introduced. And some interesting properties are discussed.

Let $\{q\}$ be a collection of intuitionistic fuzzy S^* centered (not necessarily maximal) systems of intuitionistic fuzzy S^* open sets of S. An intuitionistic fuzzy S^* topology may be defined on this collection, similar to that introduced above in $\delta(S)$. For if U is an intuitionistic fuzzy S^* open set of S, let O_U denote the collection of all intuitionistic fuzzy S^* centered systems $q \in \{q\}$ containing U as an element. All sets of the form O_U form a subbase.

Let $\delta(S)$ be an arbitrary intuitionistic fuzzy S^* extension of S. Every member $A \in \delta(S)$ in particular, A may belong to S defines a certain intuitionistic fuzzy S^* centered system in S, namely $\left\{V_\alpha^A = S \bigcap U_\alpha^A\right\}$ where U_α^A runs through all neighbourhoods of A in $\delta(S)$.

Thus, every extension of an arbitrary intuitionistic fuzzy S^* Hausdorff space S can be realized as an intuitionistic fuzzy S^* structure space of centered systems of intuitionistic fuzzy S^* open sets of S with an appropriately chosen topology.

Let $\delta_\sigma(S)$ denote the intuitionistic fuzzy S^* structure space that is obtained by introducing an intuitionistic fuzzy S^* topology into a set of intuitionistic fuzzy S^* centered systems $\left\{V_\alpha^A\right\}$ by the method mentioned above.

Lemma 4.1 For any intuitionistic fuzzy S^* extension $\delta(S)$ the intuitionistic fuzzy S^* structure space $\delta_\sigma(S)$ is an intuitionistic fuzzy S^* extension of S and intuitionistic fuzzy S^* θ-homeomorphic to $\delta(S)$.

Definition 4.1

An intuitionistic fuzzy S^* extension $\delta(S)$ is of type σ if the function i (one-to-one correspondence between the members of $\delta(S)$ and $\delta_\sigma(S)$) is an intuitionistic fuzzy S^* θ-homeomorphism.

Defenition 4.2

An intuitionistic fuzzy S^* extension $\delta(S)$ is of type τ if the set $\delta(S) \setminus S$ is discrete in the intuitionistic fuzzy S^* relative topology.

Proposition 4.1

Every intuitionistic fuzzy S^* extension of S is an intuitionistic fuzzy S^* θ-homeomorphic to some extension of type σ of the same space.

The proof follows from the fact the intuitionistic fuzzy S^* extension $\delta_\sigma(S)$ in Lemma 4.1 is of type σ.

Now let $\delta(S)$ be any intuitionistic fuzzy S^* extension. Let $\delta_\tau(S)$ denote the intuitionistic fuzzy S^* structure space obtained as follows. The members of $\delta_\tau(S)$ are those of $\delta(S)$. The intuitionistic fuzzy S^* neighbourhoods of members $A \in S$ are same as in S, but for members $A \in \delta(S) \setminus S$, the intuitionistic fuzzy S^* neighbourhoods are obtained from those of A in $\delta(S)$ by rejecting the set $\delta(S) \setminus S \bigcup A$. Clearly $\delta_\tau(S)$ an intuitionistic fuzzy S^* Hausdorff space.

Definition 4.3

Let $\left(S_1, S_1^*\right)$ and $\left(S_2, S_2^*\right)$ be two intuitionistic fuzzy $S*$ structure spaces. An intuitionistic fuzzy $S*$ structure space $\left(S_1, S_1^*\right)$ is said to be topologically embedded in another intuitionistic fuzzy $S*$ structure space $\left(S_2, S_2^*\right)$ if $\left(S_1, S_1^*\right)$ is an intuitionistic fuzzy $S*$ homeomorphism to an intuitionistic fuzzy $S*$ subspace of $\left(S_2, S_2^*\right)$.

Lemma 4.2 For any intuitionistic fuzzy $S*$ extension $\delta(S)$, the intuitionistic fuzzy $S*$ structure space $\delta_\tau(S)$ is an intuitionistic fuzzy $S*$ extension of S, intuitionistic fuzzy $S*\,\theta$-homeomorphic to $\delta(S)$ and of type τ.

Note 4.1 From Lemma 4.2 each intuitionistic fuzzy $S*$ extension $\delta(S)$ of S is associated with intuitionistic fuzzy $S*$ extensions $\delta_\sigma(S)$ and $\delta_\tau(S)$, of types σ and τ, respectively and intuitionistic fuzzy $S*\,\theta$-homeomorphic to each other and also intuitionistic fuzzy $S*\,\theta$-homeomorphic to the original intuitionistic fuzzy $S*$ extension $\delta(S)$.

Let $\mathcal{G}$ be a intuitionistic fuzzy $S*$ base of intuitionistic fuzzy $S*$ open sets in an intuitionistic fuzzy $S*$ structure space S and $\sigma_{\mathcal{G}}(S)$, the intuitionistic fuzzy $S*$ structure space whose elements are, firstly, the members of S itself and secondly, all the maximal intuitionistic fuzzy $S*$ centred systems $\{U_\alpha\}$ consisting of intuitionistic fuzzy $S*$ open sets belonging to $\mathcal{G}$, none of which contains as a subsystem all the intuitionistic fuzzy $S*$ neighbourhoods of any intuitionistic fuzzy $S*$ open set of S belonging to $\mathcal{G}$ (Clearly this condition is equivalent to the following: $\bigcap_\alpha IFS^* cl(U_\alpha) = \phi$).

An intuitionistic fuzzy $S*$ topology is defined in $\sigma_{\mathcal{G}}(S)$ as follows. If $U \in \mathcal{G}$, then O_U denotes the set of all $A \in U$ and all maximal intuitionistic fuzzy $S*$ centred system in $\sigma_{\mathcal{G}}(S)$ that contain U as an element. Since in $\sigma_{\mathcal{G}}(S)$ each member $A \in S$ can be replaced by the intuitionistic fuzzy $S*$ centred system of all its intuitionistic fuzzy $S*$ neighbourhoods belonging to $\mathcal{G}$ (with the intuitionistic fuzzy $S*$ topologization: $\{U_\alpha\} \in O_U$ if $U \in \{U_\alpha\}$). It is clear that each $\sigma_{\mathcal{G}}(S)$ is an intuitionistic fuzzy $S*$ Hausdorff extension of type σ of the original intuitionistic fuzzy $S*$ structure space S.

The intuitionistic fuzzy $S*$ structure space $\sigma_{\mathcal{G}}(S)$ may also be obtained by the following equivalent method. An intuitionistic fuzzy $S*$ centred system $\{U_\alpha\}$ of intuitionistic fuzzy $S*$ open sets of $\mathcal{G}$ is called an intuitionistic fuzzy $S*$ Hausdorff system if for every $B \in S$ not belonging to $U \in \{U_\alpha\}$ there exists a $U' \in \{U_\alpha\}$ such that $B \notin IFS^* cl(U')$. A maximal intuitionistic fuzzy $S*$ Hausdorff system (that is, one which cannot be extended while remaining an intuitionistic fuzzy S^* centred system and an intuitionistic fuzzy $S*$ Hausdorff space) is called an intuitionistic fuzzy $S*$ Hausdorff end. Then the set of all intuitionistic fuzzy $S*$ Hausdorff ends with the intuitionistic fuzzy $S*$ topology described in Section 4 is again $\sigma_{\mathcal{G}}(S)$. To establish this it is sufficient to note firstly that the set of all intuitionistic fuzzy $S*$ neighbourhoods of $\mathcal{G}$ of an arbitrary fixed intuitionistic fuzzy S^* open set A

of an intuitionistic fuzzy S^* Hausdorff end, and secondly that every maximal intuitionistic fuzzy S^* centred system not containing all the intuitionistic fuzzy S^* neighbourhoods in $\mathcal{G}$ of some member is automatically intuitionistic fuzzy S^* Hausdorff end. As the base it may be taken, in particular, the set of all intuitionistic fuzzy S^* open sets of S. An intuitionistic fuzzy S^* structure space $\sigma_{\mathcal{G}}(S)$ associated with this base will simply be denoted by $\sigma(S)$.

Proposition 4.2 An intuitionistic fuzzy S^* extension $\sigma(S)$ is an intuitionistic fuzzy S^* H-closed extension of S.

Note 4.2 Let $\mathcal{G}$ be any intuitionistic fuzzy S^* base of S. If $U \in \mathcal{G}$ then, $S \setminus IFS^*cl(U) \in \mathcal{G}$. Then $\mathcal{G}$ is called intuitionistic fuzzy S^* algebraically closed.

Remark 4.1

If $\mathcal{G}$ is an intuitionistic fuzzy S^* algebraically closed base of S, then $\sigma_{\mathcal{G}}(S)$ is an intuitionistic fuzzy S^* H-closed extension of S.

The proof is essentially the same as that of Proposition 4.2.

Note 4.3 Each intuitionistic fuzzy S^* extension $\delta(S)$ is associated with intuitionistic fuzzy S^* θ-homeomorphic extension $\sigma_\tau(S)$ of type τ, the intuitionistic fuzzy S^* structure space $\delta_\tau(S)$ which is associated with $\sigma(S)$ is denoted by $\tau(S)$ and is called an intuitionistic fuzzy S^* Katetov extension of S.

Lemma 4.3 An intuitionistic fuzzy S^* θ-continuous image of an intuitionistic fuzzy S^* H-closed space is an intuitionistic fuzzy S^* H-closed.

Remark 4.2 The intuitionistic fuzzy S^* structure space $\tau(S)$ is an intuitionistic fuzzy S^* H-closed extension of S.

The proof follows from Proposition 4.2 and Lemma 4.3.

Note 4.4 An intuitionistic fuzzy S^* structure space $\tau(S)$ has the following maximal properties.

Proposition 4.3 If $\delta(S)$ is any (not necessarily intuitionistic fuzzy S^* H-closed) intuitionistic fuzzy S^* extension of S, then there exists a subset $\tau_\delta(S) \subseteq \tau(S)$, containing S and an intuitionistic fuzzy S^* continuous function f_δ of this subset onto $\delta(S)$ such that $f_\delta(A) = A$, where $A \in S$. Here if $\delta(S)$ is an intuitionistic fuzzy S^* H-closed extension, it may be assumed that $\tau_\delta(S) = \tau(S)$.

Remark 4.3.

$\tau_{\delta\sigma}(S)$ denotes the intuitionistic fuzzy S^* structure space obtained from $\tau_\delta(S)$ by the procedure described in Section 4. It is easy to see that $\tau_{\delta\sigma}(S)$ is an intuitionistic fuzzy S^* θ- homeomorphic to a subset of the intuitionistic fuzzy S^* extension $\sigma(S)$. As $\tau_{\delta\sigma}(S)$ is an intuitionistic fuzzy S^* θ-homeomorphic to $\tau_\delta(S)$, Proposition 4.3 holds if $\tau(S)$ is replaced by $\sigma(S)$ and intuitionistic fuzzy S^* continuity by intuitionistic fuzzy S^* θ-continuity.

Remark 4.4 An intuitionistic fuzzy S^* structure space $\sigma(S)$ can be mapped intuitionistic fuzzy S^* θ-continuity onto any intuitionistic fuzzy S^* H-closed extension of S in such a way that the members of S remain fixed.

Lemma 4.4 An intuitionistic fuzzy S^* open set $\tau_\delta(S)$ is the largest subset of $\tau(S)$ that can be continuously mapped onto $\delta(S)$ in such a way that the members of S remain fixed; in other words, if a set $\tau'_\delta(S)$ is continuously mapped onto $\delta(S)$ in such a way that the members of S remain fixed, then $\tau'_\delta(S) \subseteq \tau_\delta(S)$.

Note 4.5 Thus, all intuitionistic fuzzy S^* extensions of S fall into classes, where $\delta(S)$ and $\delta'(S)$ are in the same class if and only if $\tau_\delta(S) = \tau'_\delta(S)$. All intuitionistic fuzzy S^* H -closed extensions belong to the same class, by Lemma 4.3 contains only intuitionistic fuzzy S^* H -closed extensions.

Lemma 4.5 If intuitionistic fuzzy S^* extensions $\delta(S)$ and $\gamma(S)$ are intuitionistic fuzzy S^* θ-homeomorphic, then they belong to the same class, that is $\tau_\delta(S) = \tau_\gamma(S)$.

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