

A STUDY ON FUZZY LOCALLY G_δ -CLOSED SETS

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ABSTRACT

In this paper the concept of fuzzy locally G_δ -closed sets is introduced and its interrelations with other types of locally closed sets are studied with suitable counter examples. Equivalently the interrelations of fuzzy locally G_δ continuous functions with other types of fuzzy locally continuous functions are discussed with necessary counter examples.

Key words

fuzzy locally G_δ closed sets, fuzzy locally regular closed sets, and fuzzy locally G_δ continuous functions

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INTRODUCTION AND PRELIMINARIES

The concept of fuzzy set was introduced by Zadeh [10] in his classical paper. The concept of fuzzy topological spaces was introduced and developed by Chang [6]. Fuzzy sets have applications in many fields such as information [8] and control [9]. The first step of locally closedness was done by Bourbaki [5]. Ganster and Reilly used locally closed sets in [7] to define Lc-continuity and Lc-connectedness. The concepts of r-fuzzy- G_δ - \tilde{g} -locally closed sets and fuzzy G_δ - \tilde{g} -locally continuous functions were studied by Amudhambigai, Uma and Roja [1]. In this paper the concept of fuzzy locally G_δ -closed sets is introduced and its interrelations with other types of locally closed sets are studied with suitable counter examples. Equivalently the interrelations of fuzzy locally G_δ continuous functions with other types of fuzzy locally continuous functions are discussed with necessary counter examples.

Definition 1.1 Let (X, T) be a fuzzy topological spaces. Any fuzzy set $\lambda \in I^X$ is called (i) a **fuzzy regular closed set** [2] of X if $\lambda = \text{cl}(\text{int}(\lambda))$, (ii) a **fuzzy regular open set** [2] of X if $\lambda = \text{int}(\text{cl}(\lambda))$, (iii) a **fuzzy F_σ set** [3] if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$, where $1 - \lambda_i \in T$, (iv) a **fuzzy G_δ set** [3] if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$, where $\lambda_i \in T$, (v) a **fuzzy pre-closed set** [4] if $\lambda \geq \text{cl}(\text{int}(\lambda))$, (vi) a **fuzzy α -closed set** [4] if $\lambda \geq \text{cl}(\text{int}(\text{cl}(\lambda)))$.

2.ON FUZZY LOCALLY G_δ CLOSED SETS

Definition 2.1 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally closed set** (briefly, FLCs) if $\lambda = \mu \wedge \gamma$, where μ is fuzzy open and γ is fuzzy closed. Its complement is said to be a **fuzzy locally open set**.

Definition 2.2 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally regular closed set** (briefly, FLrcs) if $\lambda = \mu \wedge \gamma$, where μ is fuzzy open and γ is fuzzy regular closed. Its complement is said to be a **fuzzy locally regular open set**.

Proposition 2.1 Every fuzzy locally regular closed set is fuzzy locally closed.

Remark 2.1 The converse of the above Proposition 3.1 need not be true.

Example 2.1 Every fuzzy locally closed set need not be fuzzy locally regular closed. Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.3, \lambda_1(b) = 0.2; \lambda_2(a) = 0.5, \lambda_2(b) = 0.8; \lambda_3(a) = 0.4, \lambda_3(b) = 0.7$. Define the fuzzy topology on X as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly, (X, T) is a fuzzy topological space. Then, for $\lambda_3 \in T, \lambda_3 \wedge 1 - \lambda_3 = (0.4, 0.3) = \lambda$ is fuzzy locally closed. But λ is not fuzzy regular closed and also λ is not a fuzzy locally regular closed set. Therefore, every fuzzy locally closed set need not be fuzzy locally regular closed.

Definition 2.3 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally G_δ closed set** (briefly, FL G_δ cs) if $\lambda = \mu \wedge \gamma$, where μ is fuzzy G_δ and γ is fuzzy closed. Its complement is said to be a **fuzzy locally F_σ open set**.

Proposition 2.2 Every fuzzy locally regular closed set is fuzzy locally G_δ closed.

Remark 2.2 The converse of the above Proposition 2.2 need not be true.

Example 2.2 Every fuzzy locally G_δ closed set need not be fuzzy locally regular closed. Let $X = [0, 1]$ and let $\lambda(n) \in I^X$ be define as $\lambda(n) = \frac{2n}{8n+2}$, where $n = 1, 2, \dots, \infty$. Define the fuzzy topology as follows: $T = \{0, \lambda(n)\}$. Now, (X, T) is a fuzzy topological space. Then, $\mu = \bigwedge_{n=1}^{\infty} \lambda(n) = \frac{1}{4}$ is fuzzy G_δ . Thus $\mu = \mu \wedge 1$ is fuzzy locally G_δ closed. But, μ is not a fuzzy locally regular closed set. Hence every fuzzy locally G_δ closed set need not be fuzzy locally regular closed.

Proposition 2.3 Every fuzzy locally closed set is fuzzy locally G_δ closed.

Remark 2.3 The converse of the above Proposition 2.3 need not be true.

Example 2.3 Every fuzzy locally G_δ closed set need not be fuzzy locally closed.

Let $X = [0, 1]$ and let $\lambda(n) \in I^X$ be define as $\lambda(n) = \frac{n}{1+7n}$, where $n = 1, 2, \dots, \infty$. Define the fuzzy topology as $T = \{0, 1, \lambda(n)\}$. Now, (X, T) is a fuzzy topological space. Then, for the fuzzy G_δ set $\mu = \bigwedge_{n=1}^{\infty} \lambda(n) = \frac{1}{7}, \mu = \mu \wedge 1$ is fuzzy locally G_δ closed. But, μ is not fuzzy locally closed.

Definition 3.4 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally pre-closed set** (briefly, FLpcs) if $\lambda = \mu \wedge \gamma$, where μ is fuzzy open and γ is fuzzy pre-closed. Its complement is said to be **fuzzy locally pre-open**.

Proposition 2.4 Every fuzzy locally closed set is fuzzy locally pre-closed.

Remark 2.4 The converse of the above Proposition 2.4 need not be true.

Example 2.4 Every fuzzy locally pre-closed set need not be fuzzy locally closed. Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.2, \lambda_1(b) = 0.3; \lambda_2(a) = 0.3, \lambda_2(b) = 0.4; \lambda_3(a) = 0.85, \lambda_3(b) = 0.75$. Define the fuzzy topology on X as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly, (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.8; \gamma(b) = 0.9$. Then γ is fuzzy pre-closed. Thus $\lambda_3 \wedge \gamma = (0.8, 0.75) = \lambda$ is fuzzy locally pre-closed. But, λ is not fuzzy locally closed.

Definition 2.5 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally α - closed set** (briefly, FL α - cs) if $\lambda = \mu \wedge \gamma$, where μ is fuzzy open and γ is fuzzy α - closed. Its complement is said to be **fuzzy locally α - open**.

Proposition 2.5 Every fuzzy locally closed set is fuzzy locally α - closed.

Remark 2.5 The converse of the above Proposition 2.5 need not be true.

Example 2.5 Every fuzzy locally α - closed set need not be fuzzy locally closed. Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.8, \lambda_1(b) = 0.85; \lambda_2(a) = 0.5, \lambda_2(b) = 0.3; \lambda_3(a) = 0.3, \lambda_3(b) = 0.1$. Define the fuzzy topology on X as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Clearly, (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.7, \gamma(b) = 0.8$. Then γ is fuzzy α - closed. Thus, for any $\lambda_i \in T, \lambda_i \wedge \gamma = \gamma$ is **fuzzy locally α - closed. But, λ is not a fuzzy locally closed set.**

Proposition 2.6 Every fuzzy locally α -closed set is fuzzy locally pre-closed.

Remark 2.6 The converse of the above Proposition 2.6 need not be true.

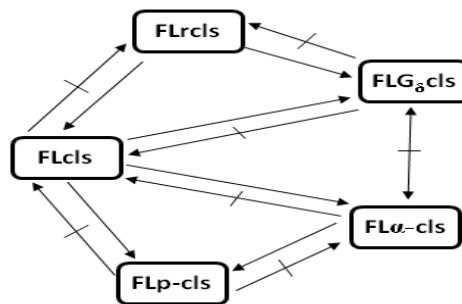
Example 2.6 Every fuzzy locally pre-closed set need not be fuzzy locally α - closed. Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.2, \lambda_1(b) = 0.3; \lambda_2(a) = 0.3, \lambda_2(b) = 0.4; \lambda_3(a) = 0.85, \lambda_3(b) = 0.75$. Define the fuzzy topology as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Clearly, (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.8, \gamma(b) = 0.9$. Then γ is fuzzy pre-closed. Thus, for $\lambda_3 \in T, \lambda_3 \wedge \gamma = \lambda = (0.8, 0.75)$ is **fuzzy locally pre-closed. But λ is not fuzzy locally α - closed.**

Remark 2.7 The notions of fuzzy locally G_δ closed sets and fuzzy locally α - closed sets are independent.

Example 2.7 Let $X = [0, 1]$ and let $\lambda(n) \in I^X$ be define as $\lambda(n) = \frac{3n}{18n+1}$, where $n = 1, 2, \dots, \infty$. Define the fuzzy topology as $T = \{ 0, 1, \lambda(n) \}$. Clearly, (X, T) is a fuzzy topological space. Then, $\mu = \bigwedge_{n=1}^{\infty} \lambda(n) = \frac{1}{6}$ is fuzzy G_δ . Thus for the fuzzy G_δ set μ and for the fuzzy closed set 1, $\mu = \mu \wedge 1 = \frac{1}{6} \wedge 1 = \frac{1}{6}$ is **fuzzy locally G_δ closed. But, μ is not fuzzy locally α - closed. Hence, every fuzzy locally G_δ closed set need not be fuzzy locally α - closed.**

Example 2.8 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.3, \lambda_1(b) = 0.1; \lambda_2(a) = 0.5, \lambda_2(b) = 0.3; \lambda_3(a) = 0.8, \lambda_3(b) = 0.85$. Define the fuzzy topology as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Clearly, (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.7, \gamma(b) = 0.8$. Then γ is fuzzy α - closed. Thus, for any $\lambda_3 \in T, \lambda_3 \wedge \gamma = \gamma$ is **fuzzy locally α - closed. But, λ is not a fuzzy locally closed set and hence not fuzzy locally G_δ closed. Therefore, every fuzzy locally α - closed set need not be fuzzy locally G_δ closed. Hence every fuzzy locally G_δ closed sets and fuzzy locally α - closed sets are of independent notions.**

Remark 2.8 Clearly the above discussions give the following implications :



3.ON FUZZY LOCALLY G_δ CONTINUOUS FUNCTIONS

Definition 3.1 Let (X, T) be a fuzzy topological space. For any fuzzy set λ of X , the **fuzzy locally pre-closure** of λ and the **fuzzy locally pre-interior** of λ are defined respectively, as $FLp-cl(\lambda) = \bigwedge \{ \mu \in I^X : \mu \geq \lambda, \mu \text{ is a fuzzy locally pre-closed set} \}$ and $FLp-int(\lambda) = \bigvee \{ \mu : \mu \leq \lambda, \mu \text{ is a fuzzy locally pre-open set} \}$.

Proposition 3.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Then for any function $f : (X, T) \rightarrow (Y, S)$ the following statements are equivalent:

- (a) f is fuzzy locally pre-continuous.
- (b) For every $\lambda \in I^X$, $f(FLpcl(\lambda)) \leq cl(f(\lambda))$.
- (c) For every $\lambda \in I^Y$, $f^{-1}(cl(\lambda)) \geq FLpcl(f^{-1}(\lambda))$.
- (d) For every $\lambda \in I^Y$, $f^{-1}(int(\lambda)) \leq FLpint(f^{-1}(\lambda))$.

Definition 3.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally continuous function** (briefly, FLCf) if for each fuzzy closed set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy locally closed.

Definition 3.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally regular continuous function** (briefly, FLrcf) if for each fuzzy closed set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy locally regular closed.

Proposition 3.2 Every fuzzy locally regular continuous function is fuzzy locally continuous.

Remark 3.1 The converse of the above Proposition 3.2 need not be true.

Example 3.1 Every fuzzy locally continuous function need not be fuzzy locally regular continuous. Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.3, \lambda_1(b) = 0.2; \lambda_2(a) = 0.5, \lambda_2(b) = 0.8; \lambda_3(a) = 0.4, \lambda_3(b) = 0.7$. Define the fuzzy topology on X as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.7, \lambda(b) = 0.6$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b, f(b) = a$. Now, $f^{-1}(1 - \lambda) = (0.4, 0.3)$ is **fuzzy locally closed in (X, T) but not fuzzy locally regular closed**. Hence **every fuzzy locally continuous function need not be fuzzy locally regular continuous**.

Definition 3.4 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally G_δ continuous function** (briefly, FLG _{δ} cf) if for each fuzzy closed set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy locally G_δ closed.

Proposition 3.3 Every fuzzy locally regular continuous function is fuzzy locally G_δ continuous.

Remark 3.2 The converse of the above Proposition 3.3 need not be true.

Example 3.2 Every fuzzy locally G_δ continuous function need not be fuzzy locally regular continuous. Let $X = [0, 1]$ and let $\lambda(n) \in I^X$ be define as $\lambda(n) = \frac{2n}{8n+2}$, where $n = 1, 2, \dots, \infty$. Let $Y = \{a\}$. Define the fuzzy topology on X as $T = \{0, 1, \lambda(n)\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ be defined as $\lambda(a) = \frac{3}{4}$. Clearly (X, T) and (Y, S) are fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be an identity function. Then, $f^{-1}(1 - \lambda) = (1 - \lambda) \in I^X$ is fuzzy locally G_δ closed. **Thus f is fuzzy locally G_δ continuous**. But, $f^{-1}(1 - \lambda)$ is not a fuzzy locally regular closed set. Hence **f is not fuzzy locally regular continuous**.

Proposition 3.4 Every fuzzy locally continuous function is fuzzy locally G_δ continuous.

Remark 3.3 The converse of the above Proposition 3.4 need not be true.

Example 3.3 Every fuzzy locally G_δ continuous function need not be fuzzy locally continuous. Let $X = [0, 1]$ and let $\lambda(n) \in I^X$ be define as $\lambda(n) = \frac{n}{1+7n}$, where $n = 1, 2, \dots, \infty$. Let $Y = \{a\}$. Define the fuzzy topology on X as $T = \{0, 1, \lambda(n)\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = \frac{6}{7}$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be an identity function.

Then, $f^{-1}(1 - \lambda) = (1 - \lambda) \in I^X$ is fuzzy locally G_δ closed. Thus **f is fuzzy locally G_δ continuous.** But

$f^{-1}(1 - \lambda)$ is not a fuzzy locally closed set. Hence **f is not fuzzy locally continuous.**

Definition 3.5 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally pre-continuous function** (briefly, FLp-cf) if for each fuzzy closed set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy locally pre-closed.

Proposition 3.5 Every fuzzy locally continuous function is fuzzy locally pre-continuous.

Remark 3.4 The converse of the above Proposition 3.5 need not be true.

Example 3.4 Every fuzzy locally pre-continuous function need not be fuzzy locally continuous. Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.2, \lambda_1(b) = 0.3; \lambda_2(a) = 0.3, \lambda_2(b) = 0.4; \lambda_3(a) = 0.85, \lambda_3(b) = 0.75$. Define the fuzzy topology on X as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.25, \lambda(b) = 0.2$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b; f(b) = a$. Now, $f^{-1}(1 - \lambda) = (0.8, 0.75)$ is fuzzy pre-closed in (X, T) . Thus, $\lambda_3 \wedge f^{-1}(1 - \lambda) = (0.8, 0.75)$ is **fuzzy locally pre-closed but not fuzzy locally closed.**

Definition 3.6 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally α -continuous function** (briefly, FL α -cf) if for each fuzzy closed set $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is fuzzy locally α -closed.

Proposition 3.6 Every fuzzy locally continuous function is fuzzy locally α -continuous.

Remark 3.5 The converse of the above Proposition 3.6 need not be true.

Example 3.5 Every fuzzy locally α -continuous function need not be fuzzy locally continuous. Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.8, \lambda_1(b) = 0.85; \lambda_2(a) = 0.5, \lambda_2(b) = 0.3; \lambda_3(a) = 0.3, \lambda_3(b) = 0.1$. Define the fuzzy topology on X as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.2, \lambda(b) = 0.3$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b; f(b) = a$. Now, $f^{-1}(1 - \lambda) = (0.7, 0.8)$ is fuzzy α -closed. Thus $\lambda_1 \wedge f^{-1}(1 - \lambda) = (0.7, 0.8)$ is **fuzzy locally α -closed but not fuzzy locally closed.**

Proposition 3.7 Every fuzzy locally α -continuous function is fuzzy locally pre-continuous.

Remark 3.6 The converse of the above Proposition 3.7 need not be true.

Example 3.6 Every fuzzy locally pre-continuous function need not be fuzzy locally α -continuous. Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.2, \lambda_1(b) = 0.3; \lambda_2(a) = 0.3, \lambda_2(b) = 0.4; \lambda_3(a) = 0.85, \lambda_3(b) = 0.75$. Define the fuzzy topology as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.1, \lambda(b) = 0.2$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b; f(b) = a$. Now, $f^{-1}(1 - \lambda) = (0.8, 0.9)$ is fuzzy pre-closed in (X, T) . Thus $\lambda_3 \wedge f^{-1}(1 - \lambda) = (0.8, 0.75)$ is **fuzzy locally pre-closed but not fuzzy locally α -closed.**

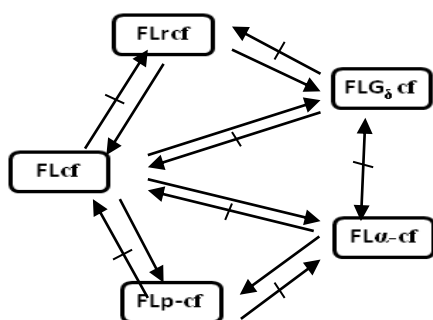
Remark 3.7 The notions of fuzzy locally G_δ continuous functions and fuzzy locally α -continuous functions are independent.

Example 3.7 Let $X = [0, 1]$ and let $\lambda(n) \in I^X$ be define as $\lambda(n) = \frac{3n}{18n+1}$, where $n = 1, 2, \dots, \infty$. Define the fuzzy topology on X as $T = \{0, 1, \lambda(n)\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ be defined as $\lambda(a) = \frac{5}{6}$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Let $f : (X, T) \rightarrow (Y, S)$ be an identity function. Then, $f^{-1}(1 - \lambda) = (1 - \lambda) \in I^X$ is fuzzy locally G_δ closed, where $f^{-1}(1 - \lambda) = \bigwedge_{n=1}^{\infty} \lambda(n)$ is fuzzy G_δ . Thus **f is fuzzy locally G_δ continuous.** But, $f^{-1}(1 - \lambda)$ is not fuzzy locally α -closed. Hence **f is not fuzzy locally α -continuous.**

Example 3.8 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as $\lambda_1(a) = 0.3, \lambda_1(b) = 0.1; \lambda_2(a) = 0.5, \lambda_2(b) = 0.3; \lambda_3(a) = 0.8, \lambda_3(b) = 0.85$. Define the fuzzy topology on X as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on Y as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.2, \lambda(b) = 0.3$. Clearly, (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b; f(b) = a$. Now, $f^{-1}(1 - \lambda) = (0.7, 0.8)$ is fuzzy α -closed. Thus $\lambda_3 \wedge f^{-1}(1 - \lambda) = (0.7, 0.8)$ is **fuzzy locally α -closed but not fuzzy locally G_δ -closed**. Hence **every fuzzy locally α -continuous function need not be fuzzy locally G_δ -continuous**.

Thus, **fuzzy locally G_δ -continuous functions and fuzzy locally α -continuous functions are of independent notions**.

Remark 3.8 Clearly the above discussions give the following implications:



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