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SOME MORE PROPERTIES OF c-SEMICONTINUOUS FUNCTIONS

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Abstract

In 1970 ,Gentry and Hoyle have defined and studied the notion of c-continuity in topological spaces. Later, Long et al and Gauld have studied some more properties of c-continuity in the literature. In 1963, N.Levine had defined and studied the concepts of semiopen sets and semicontinuity in topological spaces. In 2001, G.I.Chae et al have defined and studied the concept of c-semicontinuous functions in topology . In this paper, we study some more properties of c-semicontinuity .

1.Introduction

In 1970 , Gentry and Hoyle [11] have defined and studied the new class of functions called c-continuous functions. Latter, in 1974 & 1975 , Long et al [15 &16] have studied further properties of c-continuous functions and defined a new class of functions called c*-continuous functions in topological spaces. Again, in 1978 Gauld [10] has defined and studied some more properties of c-continuous functions via cocompact topologies. In 1963, N.Levine [14] had defined and studied the concepts of semiopen sets and semicontinuity in topological spaces. In 2001, G.I.Chae et al [4] have defined and studied the concept of c-semicontinuous functions in topology . In this paper, we study some more properties of c-semicontinuity .

2.Preliminaries

Throughout the present paper, spaces (X,τ) and (Y, σ) (or simply , X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated and $f: X \rightarrow Y$ denotes a single valued function of a space (X, τ) into a space (Y, σ). Moreover, in this paper wherever compactness is taken to mean every open cover has a finite subcover and subsets of a space are compact provided they are compact considered as subspace [cf . 11]. A be a subset of space X. The closure and the interior of A are denoted by Cl(A) and Int(A) respectively. A subset A of a space X is called regular open (resp. regular closed) if A = Int Cl(A) (resp. A = Cl Int(A)) .

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The following definitions and results are useful in the sequel :

Definition 2.1: A subset A of a space X is said to be

(i) α **-**open [23] if A $\subset$ Int(Cl(Int(A)))

(ii) semi-open [14] if A $\subset$ Cl(Int(A))

(iii) pre-open [18] if A $\subset$ Int(Cl(A))

The family of all α–open (resp. semi-open, pre-open) sets in a space X is denoted by αO(X) (resp. SO(X) PO(X.) The complement of an α-open (resp. pre-open, semiopen) set is said to α-closed [20] (resp. pre-closed [8] , semiclosed [2 & 5]).

Definition 2.2:The intersection of all semiclosed sets containing A is called the semiclosure of A and is denoted by sClA [2 & 5].

The union of all semiopen sets contained in A is called semiinterior of A and is denoted by sInt(A) [5].

Definition 2.3 [3] : A space X is called submaximal if each dense subset of X is open in X.

Lemma 2.4 [24] : A space X is submaximal iff every preopen (= dense) set is open.

Definition 2.5[25]: A space X is called extremally disconnected (in short , e.d.) space if the closure of each open set of X is open in X.

Lemma 2.6[13] : In an submaximal e.d.-space X , every semiopen set is open.

Lemma 2.7[12] : A space X is an extremally disconnected (e.d.) iff SO(X) $\subset$ PO(X).

Definition 2.8 :A function f:X $\rightarrow$ Y is said to be:

(i) precontinous [18] , if the inverse image of each open subset of Y is preopen subset in X.

(ii) semicontinuous [14] , if the inverse image of each semiopen subset of Y is an open subset in X.

(iii) strongly semicontinuous [1] , if the inverse image of each semiopen subset of Y is an open subset in X.

(iv) irresolute [6] , if the inverse image of each semiopen subset of Y is semiopen subset in X., equivalently, if the inverse image of each semiclosed subset of Y is semiclosed subset in X.

Definition 2.5[21] : A function f:X $\rightarrow$ Y is said to be M-preopen(resp. M-preclosed) if the image of each preopen (resp. preclosed) subset of X is preopen (resp. preclosed) subset in Y.

Definition 2.6 [7] : A function f : X Y is said to be presemiopen , if the image of each semiopen subset of X is semiopen in Y.

Definition 2.6 [9] : A function f : X Y is said to be presemiclosed , if the image of each semiclosed subset of X is semiclosed in Y.

Definition 2.6[11] : A function $f : X \to Y$ is said to be c-continuous if for each $x \in X$ and each open set $V \subset Y$ containing f (x) and having compact complement , there exists an open set U containing x such that $f(U) \subset V$.

Theorem 2.7[11,Th. 1] : Let $f : X \to Y$ be a function .Then the following statements are equivalent :

(i)f is c-continuous.

(ii)If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is open subset of X.

These statements are implied by :

(iii)If F is a compact subset of Y , then $f^{-1}(F)$ is closed subset of X and, moreover , if Y is Hausdorff, then all the above statements : (i)-(iii) are equivalent.

Theorem 2.8 : Let $f : X \to Y$ be a function . Then, f is c-continuous iff :

(i)The inverse image of each open subset of Y having compact complement is open in X [11].

(ii)The inverse image of each closed compact subset of Y is closed in X [15].

3. Properties of c-semicontinuous functions

We, recall the following.

Definition 3.1 [4] : A function $f : X \to Y$ is said to be c-semicontinuous if for each $x \in X$ and each open set V $\subset$ Y containing f (x) and having compact complement , there exists an semiopen set U containing x such that $f(U) \subset V$.

Clearly, every c-continuous function is c-semicontinuous but converse is true for a c-semicontinuous function $f : X \to Y$ with a submaximal e.d.-space X .

We, prove the following .

Theorem 3.2[4] : Let $f : X \to Y$ be a function .Then the following statements are equivalent :

(i) f is c-semicontinuous.

(ii) If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is semiopen subset of X.

These statements are implied by :

(iii) If F is a compact subset of Y , then $f^{-1}(F)$ is semiclosed subset of X and, moreover , if Y is Hausdorff, then all the above statements : (i)-(iii) are equivalent.

Proof follows by Theorem 2.7 and 2.8 above.

Easy proof of the following is omitted.

Lemma 3.3 : A function $f : X \to Y$ is said to be c-semicontinuous if the inverse image of each open subset of Y having compact complement is semiopen in X.

Lemma 3.4 : A function $f : X \to Y$ is said to be c-semicontinuous if the inverse image of each closed compact subset of Y is semiclosed in X.

We, recall the following.

Lemma 3.5[19] : If A is a preopen subset of X and V is a preopen subset of X , then $A \cap V$ is a semiopen subset in the subspace (A , τ/A) .

Next, we prove the following.

Theorem 3.6 : If $f : X \to Y$ is c-semicontinuous function and A be a preopen subset of X , then $f/A : A \to Y$ is also c –semicontinuous .

Easy proof of the Theorem follows by Lemma – 3.5 above.

Theorem 3.9 : If $f : X \to Y$ is an irresolute and $g : Y\to Z$ is c- semicontinuous, then gof is c-semicontinuous.

Proof. Let U be an open subset of Z having compact complement. Then, $g^{-1}(U)$ is semiopen set in Y , since g is c-semicontinuous. Again, as f is an irresolute and $g^{-1}(U)$ is semiopen subset of Y , $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is semiopen subset in X. This shows that gof is c-semicontinuous function.

Theorem 3.10: If $f : X \to Y$ is strongly-semicontinuous and $g: Y\to Z$ is c-semicontinuous, then gof is c-continuous function.

Proof follows from Theorem-3.6.

Theorem 3.11 : Let $f : X \to Y$ be either presemiopen or presemiclosed surjection and let $g : Y\to Z$ be any function such that gof is c-semicontinuous.Then, g is c-semicontinuous .

Proof : Suppose f is presemiopen (resp.presemiclosed) and V be an open subset with compact complement (resp. V be a closed compact subset) in Z . Since gof is c-semicontinuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semiopen (resp. semiclosed) subset in X . Since f is presemiopen (resp. presemiclosed) and surjective , $f (f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semiopen (resp. semiclosed) set in Y and consequently, g is c-semicontinuous function.

We, define the following.

Definition 3.12: A function $f : X \to Y$ is said to be s-open (resp. s-closed) if the image of each semiopen (resp. semiclosed) subset of X is open (resp. closed) subset in Y.

Theorem 3.12 : Let $f : X \to Y$ be either s-open or s-closed surjection and let $g : Y\to Z$ be any function such that gof is c-semicontinuous.Then, g is c-continuous.

Proof follows by Theorem -3.8 above.

We, recall the following.

Definition 3.14 [22]: Let $f : X \to Y$ be a function .Then the following statements are equivalent :

(i) f is c-precontinuous.

(ii) If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is preopen subset of X.

It is clear that in an e.d.-space X , every c-semicontinuous function is c-precontinuous .

Next, we prove the following.

Theorem 3.15 : Let $f : X \to Y$ be a M-preopen surjection with an e.d.-space X and let $g : Y \to Z$ be any function such that gof is c-semicontinuous.Then, g is c-precontinuous .

Proof : Suppose f is M-preopen surjective function with an e.d.-space X and V be an open subset with compact complement in Z . Since gof is c-semicontinuous , $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semiopen subset in X . Since a space X is an e.d.-space and hence $(gof)^{-1}(V)$ is preopen subset in X. Also, as f is M-preopen and surjective function , $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is preopen set in Y and consequently, g is c- precontinuous function.

In view of the fact that an arbitrary union of preopen (resp. semiopen) sets is preopen (resp. semiopen) , we have the following [14&18].

Theorem 3.16 : If X and Y are two topological spaces and $X = A \cup B$, where A and B are preopen subsets of X and $f : X \to Y$ is a function such that f|A and f|B are c-semicontinuous , then f is c-semicontinuous.

Proof : Assume that A and B are preopen subsets in X. Let U be an open subset of Y with compact complement. Then, we hav $f^{-1}(U) = (f|A)^{-1}(U) \cup (f|B)^{-1}(U)$, each of which is semiopen by Theorem- 3.5. Thus, $f^{-1}(U)$ is semiopen in X and hence f is c-semicontinuous.

In view of Definition -2.3 and Lemma-2.4 , we give the following.

Theorem 3. 17 : Let X be a submaximal e.d.- space and let Y be a locally compact regular space. If , $f : X \to Y$ is c-precontinuous then f is continuous.

Recall that a space X is called semi-T_1 [17] if, for x , y $\in$X such that $x \neq y$, there exist semiopen sets U and V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. Also, it is proved that in [17] a semi-T_1 space every singleton set is semiclosed.

In view of the above result , we give the following.

Theorem 3.18 : Let $f : X \to Y$ be c-semicontinuous and injective. If Y is T_1 , then X is semi-T_1.

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