ON c-PRECONTINUOUS FUNCTIONS

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Abstract

In 1970 ,Gentry and Hoyle have defined and studied the notion of c-continuity in topological spaces. Later, Long et al and Gauld have studied some more properties of c-continuity in the literature. In 1982,Mashhour et al have defined and studied the concepts of preopen sets and precontinuity in topology .In this paper, we define and study the concept of c-precontinuity . Also, we characterize their basic properties .

1.Introduction

In 1970 , Gentry and Hoyle [10] have defined and studied the new class of functions called c-continuous functions. Latter, in 1974 & 1975 , Long et al [14 &15] have studied further properties of c-continuous functions and defined a new class of functions called c*-continuous functions in topological spaces. Again, in 1978 Gauld [9] has defined and studied some more properties of c-continuous functions via cocompact topologies. In 1982,Mashhour et al [16] have defined and studied the concepts of preopen sets and precontinuity in topology .In this paper, we define and study the concept of c-precontinuity . Also, we characterize their basic properties in topological spaces.

2.Preliminaries

Throughout the present paper, spaces (X,τ) and (Y, σ) (or simply , X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated and f : X $\rightarrow$ Y denotes a single valued function of a space (X, τ) in to a space (Y, σ). Moreover, in this paper wherever compactness is taken to mean every open cover has a finite subcover and subsets of a space are compact provided they are compact considered as subspace

[cf . 10]. A be a subset of space X. The closure and the interior of A are denoted by Cl(A) and Int(A) respectively. A subset A of a space X is called regular open (resp. regular closed) if A = Int Cl(A) (resp. A = Cl Int(A)) .

The following definitions and results are useful in the sequel :

Definition 2.1: A subset A of a space X is said to be

(i) α -open [23] if A $\subset$ Int(Cl(Int(A)))

(ii) semi-open [13] if A $\subset$ Cl(Int(A))

(iii) pre-open [16] if A $\subset$ Int(Cl(A))

The family of all α –open (resp. semi-open, pre-open) sets in a space X is denoted by α O(X) (resp. SO(X) PO(X.) The complement of an α -open (resp. pre-open) set is said to α -closed [18] (resp. pre-closed [8]).

Definition 2.2:The intersection of all preclosed sets containing A is called the preclosure of A and is denoted by pClA [8].

The union of all pre-open sets contained in A is called pre-interior of A and is denoted by pInt(A) [19].

Definition 2.3 [3&7] : A space X is called submaximal if each dense subset of X is open in X.

Lemma 2.4 [6 & 24] : A space X   is submaximal iff every preopen (= dense) set is open.

Definition 2.3: A function f:X $\rightarrow$ Y is said to be:

(i)   precontinous [16] , if the inverse image of each open subset of Y is preopen subset in X.
(ii)   p-continuous [20] , if the inverse image of each preopen subset of Y is an open subset in X.
(iii)   M-precontinuous [19] , if the inverse image of each preopen subset of Y is preopen subset in X., equivalently, if the inverse image of each preclosed subset of Y is preclosed subset in X.

Definition 2.4 [21] : A function f : X $\rightarrow$ Y is said to be p-open (resp. p-closed) if the image of each preopen (resp. preclosed) subset of X is open (resp. closed) subset in Y.

Definition 2.5[19] : A function f:X $\rightarrow$ Y is said to be M-preopen(resp. M-preclosed) if the image of each preopen (resp. preclosed) subset of X is preopen (resp. preclosed) subset in Y.

Definition 2.6 [5] : A function f : X Y is said to be presemiopen , if the image of each semiopen subset of X is semiopen in Y.

Definition 2.6[10] : A function f : X $\rightarrow$ Y is said to be c-continuous if for each x $\in$ X and each open set V $\subset$ Y containing f (x) and having compact complement , there exists an open set U containing x such that f(U) $\subset$ V.

Theorem 2.7[10,Th. 1] : Let f : X→Y be a function .Then the following statements are equivalent :

(i)f is c-continuous.

(ii)If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is open subset of X.

These statements are implied by :

(iii)If F is a compact subset of Y , then $f^{-1}(F)$ is closed subset of X and, moreover , if Y is Hausdorff, then all the above statements : (i)-(iii) are equivalent.

Theorem 2.8 : Let f : X → Y be a function . Then, f is c-continuous iff :

(i)The inverse image of each open subset of Y having compact complement is open in X [10].

(ii)The inverse image of each closed compact subset of Y is closed in X [14].

3. Properties of c-precontinuous functions

We, define the following.

Definition 3.1 : A function f : X → Y is said to be c-precontinuous if for each x ∈ X and each open set V ⊂ Y containing f (x) and having compact complement , there exists an preopen set U containing x such that f(U) ⊂ V.

Clearly, every c-continuous function is c-precontinuous but converse is true for a c-precontinuous function f : X → Y with a submaximal space X .

We, prove the following .

Theorem 3.2 : Let f : X→Y be a function .Then the following statements are equivalent :

(i)   f is c-precontinuous.

(ii)   If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is preopen subset of X.

These statements are implied by :

(iii)   If F is a compact subset of Y , then $f^{-1}(F)$ is preclosed subset of X and, moreover , if Y is Hausdorff, then all the above statements : (i)-(iii) are equivalent.

Proof follows by Theorem 2.7 and 2.8 above.

Easy proof of the following is omitted.

Lemma 3.3 : A function f : X → Y is said to be c-precontinuous if the inverse image of each open subset of Y having compact complement is preopen in X.

ON c-PRECONTINUOUS FUNCTIONS

Lemma 3.4 : A function f : X → Y is said to be c-precontinuous if the inverse image of each closed compact subset of Y is preclosed in X.

We, recall the following.

Lemma 3.5[8] : If A is an α-open subset of X and V is a preopen subset of X , then A ∩ V is a preopen subset in the subspace (A , τ/A) .

Next, we prove the following.

Theorem 3.6 : If f : X → Y is c-precontinuous function and A be an α-open subset of X , then f/A : A → Y is also c –precontinuous .

Easy proof of the Theorem follows by Lemma – 3.5 above.

Lemma 3.7 [17] : If V ∈ PO(X) and U ∈ SO(X) , then U ∩ V ∈ PO(U).

Now, we give the following.

Theorem 3.8 : If f : X → Y is c-precontinuous function and A be an semiopen subset of X , then f/A : A → Y is also c –precontinuous .

Easy proof of the Theorem follows by Lemma – 3.7 above.

Theorem 3.9 : If f : X → Y is M-precontinuous and g : Y→ Z is c-precontinuous, then gof is c-precontinuous.

Proof. Let U be an open subset of Z having compact complement. Then, $g^{-1}(U)$ is preopen set in Y , since g is c-precontinuous. Again, as f is M-precontinuous and $g^{-1}(U)$ is preopen subset of Y , $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is preopen subset in X. This shows that gof is c-precontinuous function.

Theorem 3.10: If f : X →Y is p-continuous and g: Y→ Z is c-precontinuous, then gof is c-continuous function.

Proof follows from Theorem-3.6.

Theorem 3.11 : Let f : X → Y be either M-preopen or M-preclosed surjection and let g : Y→ Z be any function such that gof is c-precontinuous.Then, g is c-precontinuous .

Proof : Suppose f is M-preopen (resp. M-preclosed) and V be an open subset with compact complement (resp. V be a closed compact subset) in Z . Since gof is c-precontinuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is preopen (resp. preclosed) subset in X . Since f is M-preopen (resp. M-preclosed) and surjective , $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is preopen (resp. preclosed) set in Y and consequently, g is c-precontinuous function.

Theorem 3.12 : Let f : X → Y be either p-open or p-closed surjection and let g : Y→ Z be any function such that gof is c-precontinuous.Then, g is c-continuous.

Proof follows by Theorem -3.8 above.

We, recall the following.

Definition 3.13 [1]: A space X is said to be PS-space if PO(X) ⊂ SO(X) (i.e., if every preopen set of X is semiopen in it).

Definition 3.14 [4]: Let f : X→Y be a function .Then the following statements are equivalent :

    (i)   f is c-semicontinuous.

    (ii)   If V is an open subset of Y with compact complement , then $f^{-1}(V)$ is semiopen subset of X.

It is clear that in a PS-space X , every c-precontinuous function is c-semicontinuous .

Next, we prove the following.

Theorem 3.15 : Let f : X → Y be a presemiopen surjection with a PS-space X and let g : Y→ Z be any function such that gof is c-precontinuous.Then, g is c-semicontinuous .

Proof : Suppose f is presemiopen surjective function with a PS-space X and V be an open subset with compact complement in Z . Since gof is c-precontinuous , $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is preopen subset in X . Since a space X is PS-space and hence $(gof)^{-1}(V)$ is semiopen subset in X. Also, as f is presemiopen and surjective function , f ($f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semiopen set in Y and consequently, g is c- semicontinuous function.

In view of the fact that an arbitrary union of preopen (resp. α-open) sets is preopen (resp. α-open) , we have the following [16 & 18].

Theorem 3.16 : If X and Y are two topological spaces and X = A ∪ B, where A and B are preopen subsets of X and f : X → Y is a function such that f|A and f|B are c-precontinuous , then f is c-precontinuous.

Proof : Assume that A and B are α-open subsets in X. Let U be an open subset of Y with compact complement. Then, we hav $f^{-1}(U) = (f|A)^{-1}(U) ∪ (f|B)^{-1}(U)$, each of which is preopen by Theorem- 3.5. Thus, $f^{-1}(U)$ is preopen in X and hence f is c-precontinuous.

In view of Definition -2.3 and Lemma-2.4 , we give the following.

Theorem 3. 17 : Let X be a submaximal space and let Y be a locally compact regular space. If , f : X → Y is c-precontinuous then f is continuous.

Recall that a space X is called pre-T_1 [12] if, for x , y $\in$X such that   x $\neq$ y, there exist preopen sets U and V such that x $\in$ U   , y $\notin$ U   and y $\in$ V , x$\notin$ V. Also, it is proved that in [12 &22] a pre-T_1 space every singleton set is preclosed.

In view of the above result , we give the following.

Theorem 3.18 : Let f : X$\rightarrow$Y be c-precontinuous and injective. If Y is T_1 , then X is pre-$T_1.$

We, recall the following.

Definition 3.19[11]: Let f : X $\rightarrow$ Y be a function.Then, G(f) ={(x,f(x)) | x $\in$X } is called the graph of f and the function g(f) : X $\rightarrow$ X x Y defined as g(f)(x) = (x,f(x)) for each x $\in$X is called the graph function of f .

Definition 3.20 [2] : A function f : X $\rightarrow$ Y is said to have preclosed graph, if for each pair y $\neq$ f(x) , there exists a preopen set V of Y containing y and a preopen set U containing x $\in$X such that f(U) $\cap$ V = $\varnothing$.

Next, we prove the following.

Theorem 3.21: Let Y be a locally compact Hausdorff space.Then, c-precontinuous function   f : X $\rightarrow$ Y has a preclosed graph.

Proof : Let (x,y) $\notin$ G(f) . Then, y $\neq$ f(x) . As , Y is locally compact Hausdorff space, so there exists an open set V containing y such that f(x) $\notin$ Cl(V) , where Cl(V) is compact in Y. Now, f(x) $\in$Y \ Cl(V) , i.e., x $\in$ f^{-1}(Y \ Cl(V)). Since f is c-precontinuous , there exists a preopen set U containing x such that   x $\in$ U $\subset$ f^{-1}(Y \ Cl(V)). Thus, f(U) $\cap$ Cl(V) = $\varnothing$, i.e., f (U) $\cap$ V = $\varnothing$. This shows that f has a preclosed graph.

Theorem 3.22 : Let f : X $\rightarrow$ Y be c-precontinuous. Then, the graph function g(f) : X $\rightarrow$ X x Y is c-precontinuous.

Proof : Let U x V be any open subset in X x Y having compact complement W of X x Y. Then, we have to show that (g(f))$^{-1}$(U x V) is preopen set in X. Let   W= X x Y \ (U x V) = (X\U) x Y $\cup$ X x (Y\V) , in which X x (Y\V) being the closed subset of W must also be compact. Since P_Y : X x Y $\rightarrow$ Y being the projection, which is continuous, so P_Y (X x (Y \ V)) = Y\V is compact in Y. Thus, f $^{-1}$(V) is preopen set in X. Since f is c-precontinuous , (g(f))$^{-1}$(U x V)=U$\cap$f^{-1}(V) , which is preopen as the intersection of an open set and a preopen set is again preopen .Therefore, g(f) is c-precontinuous.

Theorem 3.23: Let X be compact Hausdorff space. If g(f) : X $\rightarrow$X x Y is c-precontinuous, then the function f : X $\rightarrow$Y is c-precontinuous.

Proof : Let V any open set containing f(x) having compact complement.Then, we have to prove that f^{-1}(V) is preopen in X : Consider X x V which is open in X x Y where X x Y \(X x V) = X x (Y\V) is compact, and g(f) is c-precontinuous and hence (g(f))$^{-1}$(X x V) = f^{-1}(V) which is preopen in X.This shows that f is c-precontinuous.

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