A STUDY ON FUZZY LOCALLY $b$-CLOSED SETS

G. VASUKI & T.PREMAKUMARI

Associate Professor of Mathematics
Dr. B.AMUDHAMBIGAI
Assistant Professor of Mathematics
Department of Mathematics
Sri Sarada College for Women, Salem-16
Tamil Nadu, India.

T.PREMAKUMARI
Sri Sarada College for Women, Salem-16
Tamil Nadu, India.

ABSTRACT

In this paper the concept of fuzzy locally $b$-closed sets is introduced and its inter relations with other types of locally closed sets are studied with suitable counter examples. Equivalently the inter relations of fuzzy locally $b$-continuous functions with other types of fuzzy locally continuous functions are discussed with necessary counter examples. Also the concepts of fuzzy locally $b$-compact spaces, fuzzy locally $b$-Lindelof spaces and fuzzy locally $b$-closed compact spaces are introduced and some of their characterizations and properties are established.

Key words
fuzzy locally $b$-closed sets, fuzzy locally $b$-continuous functions, fuzzy locally $b$-compact spaces and fuzzy locally $b$-Lindelof spaces

1. INTRODUCTION AND PRELIMINARIES

1.1 Introduction

The concept of fuzzy sets was introduced by Zadeh [11] in his classical paper. Fuzzy sets have applications in many fields such as information [9] and control [10]. The first step of locally closedness was done by Bourbaki [5]. Ganster and Reilly used locally closed sets in [8] to define $Lc$ - continuity and $Lc$ - compactness. The concepts of $r$ - fuzzy $G_{\delta}$ - locally closed sets and fuzzy $G_{\delta}$ - locally continuous functions were studied by Amudhambigai, Uma and Roja [1]. The concepts of fuzzy slightly $\beta$-continuity, fuzzy $\beta$-Lindelof, fuzzy mildly compact and fuzzy countably $\beta$-closed compact were introduced by Erdal Ekici [7]. In this paper the concept of fuzzy locally $b$-closed sets is introduced and its inter relations with other types of locally closed sets are studied with suitable counter examples. Equivalently the inter relations of fuzzy locally $b$-continuous functions with other types of fuzzy locally continuous functions are discussed with necessary counter examples. Also the concepts of fuzzy locally $b$-compact spaces, fuzzy locally $b$-Lindelof spaces and
A STUDY ON FUZZY LOCALLY b-CLOSED SETS

fuzzy locally b-closed compact spaces are introduced and some of their characterizations and properties are established.

1.2 PRELIMINARIES

Definition 1.2.2 [2,6] A fuzzy topological space \((X, T)\) is said to be fuzzy compact if every fuzzy open cover of \((X, T)\) has a finite subcover.

Definition 1.2.4 [4] Any fuzzy \(\alpha\)-closed set (briefly, \(F\alpha\)-cls) if \(\lambda \geq cl (int (\lambda))\).

Definition 1.2.7 [3] Any fuzzy set \(\lambda \in \mathcal{I}^X\) in a fuzzy topological space \((X, T)\) is said to be fuzzy \(\beta\)-closed set (briefly, \(F\beta\)-cls) if \(\lambda \geq cl (int (\lambda))\).

Definition 1.2.8 [6] Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces. Any function \(f : (X, T) \rightarrow (Y, S)\) is said to be a fuzzy continuous function if \(f^{-1}(\lambda)\) is fuzzy open in \((X, T)\) for each fuzzy open set \(\lambda\) in \((Y, S)\).

Definition 1.2.9 [7] A fuzzy topological space \((X, T)\) is said to be fuzzy \(\beta\)-compact if every fuzzy \(\beta\)-open cover of \((X, T)\) has a finite subcover.

(a) fuzzy countably \(\beta\)-compact if every fuzzy \(\beta\)-open countably cover of \((X, T)\) has a finite subcover.

(b) fuzzy \(\beta\)-Lindelof if every fuzzy \(\beta\)-open cover of \((X, T)\) has a countable subcover.

(c) fuzzy mildly compact if every fuzzy clopen cover of \((X, T)\) has a finite subcover.

(d) fuzzy mildly countably compact if every fuzzy clopen countably cover of \((X, T)\) has a finite subcover.

(e) fuzzy mildly Lindelof if every fuzzy clopen cover of \((X, T)\) has a countable subcover.

2. A STUDY ON FUZZY LOCALLY b-CLOSED SETS

Definition 2.1 Let \((X, T)\) be a fuzzy topological space. Any \(\lambda \in \mathcal{I}^X\) is called a fuzzy locally closed set (briefly, \(FL\)cls) if \(\lambda = \mu \land \gamma\), where \(\mu\) is a fuzzy open set and \(\gamma\) is a fuzzy closed set. Its complement is called a fuzzy locally open set.

Definition 2.2 Let \((X, T)\) be a fuzzy topological space. Any \(\lambda \in \mathcal{I}^X\) is called a fuzzy locally \(\alpha\)-closed set (briefly, \(FL\alpha\)-cls) if \(\lambda = \mu \land \gamma\), where \(\mu\) is a fuzzy open set and \(\gamma\) is a fuzzy \(\alpha\)-closed set. Its complement is called a fuzzy locally \(\alpha\)-open set.

Definition 2.3 Let \((X, T)\) be a fuzzy topological space. Any \(\lambda \in \mathcal{I}^X\) is called a fuzzy locally semi-closed set (briefly, \(FL\)scls) if \(\lambda = \mu \land \gamma\), where \(\mu\) is a fuzzy open set and \(\gamma\) is a fuzzy semi-closed set. Its complement is called a fuzzy locally semi-open set.

Definition 2.4 Let \((X, T)\) be a fuzzy topological space. Any \(\lambda \in \mathcal{I}^X\) is called a fuzzy locally pre-closed set (briefly, \(FL\)pcls) if \(\lambda = \mu \land \gamma\), where \(\mu\) is a fuzzy open set and \(\gamma\) is a fuzzy pre-closed set. Its complement is called a fuzzy locally pre-open set.
Definition 2.5 Let \((X, T)\) be a fuzzy topological space. Any \(\lambda \in I^X\) is called a fuzzy locally b-closed set (briefly, FLb-cl) if \(\lambda = \mu \land \gamma\), where \(\mu\) is a fuzzy open set and \(\gamma\) is a fuzzy b-closed set. Its complement is called a fuzzy locally b-open set.

Proposition 2.1 Every fuzzy locally closed set is fuzzy locally \(\alpha\)-closed.

Remark 2.1 The converse of the above Proposition 2.1 need not be true.

Example 2.1 Let \(X = \{a, b\}\) and \(\lambda_1, \lambda_2, \lambda_3 \in I^X\) be defined as follows: \(\lambda_1(a) = 0.5, \lambda_1(b) = 0.4; \lambda_2(a) = 0.5, \lambda_2(b) = 0.2; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9\). Define the fuzzy topology \(T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}\). Clearly \((X, T)\) is a fuzzy topological space. Let \(\gamma \in I^X\) be defined as \(\gamma(a) = 0.5, \gamma(b) = 0.7\). Then \(\gamma\) is fuzzy \(\alpha\)-closed. Thus, for \(\lambda_3 \in T, \lambda_3 \land \gamma = (0.5, 0.7) = \lambda\) is fuzzy locally \(\alpha\)-closed. But, \(\lambda\) is not a fuzzy locally closed set.

Proposition 2.2 Every fuzzy locally \(\alpha\)-closed set is fuzzy locally semi-closed.

Remark 2.2 The converse of the above Proposition 2.2 need not be true.

Example 2.2 Let \(X = \{a, b\}\) and \(\lambda_1, \lambda_2, \lambda_3 \in I^X\) be defined as follows: \(\lambda_1(a) = 0.3, \lambda_1(b) = 0.4; \lambda_2(a) = 0.4, \lambda_2(b) = 0.4; \lambda_3(a) = 0.8, \lambda_3(b) = 0.9\). Define the fuzzy topology \(T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}\). Clearly \((X, T)\) is a fuzzy topological space. Let \(\gamma \in I^X\) be defined as \(\gamma(a) = 0.6, \gamma(b) = 0.5\). Then \(\gamma\) is fuzzy semi-closed. Thus, for \(\lambda_3 \in T, \lambda_3 \land \gamma = (0.6, 0.5) = \lambda\) is fuzzy locally semi-closed. But, \(\lambda\) is not a fuzzy locally \(\alpha\)-closed set.

Proposition 2.3 Every fuzzy locally semi-closed set is fuzzy locally b-closed.

Remark 2.3 The converse of the above Proposition 2.3 need not be true.

Example 2.3 Let \(X = \{a, b\}\) and \(\lambda_1, \lambda_2, \lambda_3 \in I^X\) be defined as follows: \(\lambda_1(a) = 0.6, \lambda_1(b) = 0.8; \lambda_2(a) = 0.5, \lambda_2(b) = 0.4; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9\). Define the fuzzy topology \(T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}\). Clearly \((X, T)\) is a fuzzy topological space. Let \(\gamma \in I^X\) be defined as \(\gamma(a) = 0.6, \gamma(b) = 0.7\). Then \(\gamma\) is fuzzy b-closed. Thus, for \(\lambda_3 \in T\) and for the fuzzy b-closed set \(\gamma, \lambda_3 \land \gamma = (0.6, 0.7) = \lambda\) is fuzzy locally b-closed. But, \(\lambda\) is not a fuzzy locally semi-closed set.

Proposition 2.4 Every fuzzy locally closed set is fuzzy locally pre-closed.

Remark 2.4 The converse of the above Proposition 2.4 need not be true.

Example 2.4 Let \(X = \{a, b\}\) and \(\lambda_1, \lambda_2, \lambda_3 \in I^X\) be defined as follows: \(\lambda_1(a) = 0.5, \lambda_1(b) = 0.6; \lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.9, \lambda_3(b) = 0.8\). Define the fuzzy topology \(T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}\). Clearly \((X, T)\) is a fuzzy topological space. Let \(\gamma \in I^X\) be defined as \(\gamma(a) = 0.7, \gamma(b) = 0.5\). Then \(\gamma\) is fuzzy pre-closed. Thus, for \(\lambda_3 \in T\) and for the fuzzy pre-closed set \(\gamma, \lambda_3 \land \gamma = (0.7, 0.5) = \lambda\) is fuzzy locally pre-closed. But, \(\lambda\) is not a fuzzy locally closed set.

Proposition 2.5 Every fuzzy locally pre-closed set is fuzzy locally b-closed.

Remark 2.5 The converse of the above Proposition 2.5 need not be true.
Example 2.5 Let $X = \{a, b\}$ and $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.3, \lambda_1(b) = 0.2; \lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.5, \lambda_3(b) = 0.9$. Define the fuzzy topology $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly $(X, T)$ is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.7, \gamma(b) = 0.5$. Then $\gamma$ is fuzzy $b$-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy $b$-closed set $\gamma$, $\lambda_3 \land \gamma = (0.5, 0.5) = \lambda$ is fuzzy locally $b$-closed. But, $\lambda$ is not a fuzzy locally pre-closed set.

Remark 2.6 Clearly the above discussions give the following implications:

### 4. A STUDY ON FUZZY LOCALLY b-CONTINUOUS FUNCTION

**Definition 4.1** Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally continuous function** (briefly, $FLcf$) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally closed and for each fuzzy closed set $\lambda \in I^Y$.

**Definition 4.2** Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally $\alpha$-continuous function** (briefly, $FL\alpha-cf$) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally $\alpha$-closed for each fuzzy closed set $\lambda \in I^Y$.

**Definition 4.3** Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. Any $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally semi-continuous function** (briefly, $FLs-cf$) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally semi-closed and for each fuzzy closed set $\lambda \in I^Y$.

**Definition 4.4** Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally pre-continuous function** (briefly, $FLp-cf$) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally pre-closed and for each fuzzy closed set $\lambda \in I^Y$.

**Definition 4.5** Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally b-continuous function** (briefly, $FLb-cf$) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally b-closed and for each fuzzy closed set $\lambda \in I^Y$.

**Definition 4.6** Let $(X, T)$ be a fuzzy topological space. For a fuzzy set $\lambda$ of $X$, the **fuzzy locally $\alpha$-closure** (briefly, $FL\alpha-cl$) and **fuzzy locally $\alpha$-interior** (briefly, $FL\alpha-int$) of $\lambda$ are defined respectively, as $FL\alpha-cl(\lambda) = \land \{\mu \in I^X : \mu \geq \lambda; \mu$ is fuzzy locally $\alpha$-closed set $\}$ and
**Proposition 4.1** Let $(X, T)$ and $(Y, S)$ be any two fuzzy topological spaces. Then for any function $f: (X, T) \to (Y, S)$ the following statements are equivalent:

(a) $f$ is fuzzy locally $\alpha$-continuous.

(b) For every $\lambda \in I^X$, $f(FL\alpha - \text{cl}(\lambda)) \subseteq \text{cl}(f(\lambda))$.

(c) For every $\lambda \in I^Y$, $f^{-1}(\text{cl}(\lambda)) \supseteq FL\alpha - \text{cl}(f^{-1}(\lambda))$.

(d) For every $\lambda \in I^Y$, $f^{-1}(\text{int}(\lambda)) \subseteq FL\alpha - \text{int}(f^{-1}(\lambda))$.

**Proposition 4.2** Every fuzzy locally continuous function is fuzzy locally $\alpha$-continuous.

**Remark 4.1** The converse of the above Proposition 4.2 need not be true.

**Example 4.1** Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows:

$\lambda_1(a) = 0.5, \lambda_1(b) = 0.4; \lambda_2(a) = 0.5, \lambda_2(b) = 0.2; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9$. Define the fuzzy topology on $X$ as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on $Y$ as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.3, \lambda(b) = 0.5$. Clearly $(X, T)$ and $(Y, S)$ are fuzzy topological spaces. Define $f: (X, T) \to (Y, S)$ as $f(a) = b, f(b) = a$. Then, $f^{-1}(1-\lambda) = (0.5, 0.7)$ is fuzzy $\alpha$-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy $\alpha$-closed set $f^{-1}(1-\lambda), \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally $\alpha$-closed but not fuzzy locally closed in $(X, T)$.

Therefore, every fuzzy locally $\alpha$-continuous function need not be fuzzy locally continuous.

**Proposition 4.3** Every fuzzy locally $\alpha$-continuous function is fuzzy locally semi-continuous.

**Remark 4.2** The converse of the above Proposition 4.3 need not be true.

**Example 4.2** Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows:

$\lambda_1(a) = 0.3, \lambda_1(b) = 0.4; \lambda_2(a) = 0.4, \lambda_2(b) = 0.4; \lambda_3(a) = 0.8, \lambda_3(b) = 0.9$. Define the fuzzy topology on $X$ as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on $Y$ as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.5, \lambda(b) = 0.4$. Clearly $(X, T)$ and $(Y, S)$ are fuzzy topological spaces. Define $f: (X, T) \to (Y, S)$ as $f(a) = b, f(b) = a$. Then, $f^{-1}(1-\lambda) = (0.6, 0.5)$ is fuzzy semi-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy semi-closed set $f^{-1}(1-\lambda), \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally semi-closed but not fuzzy locally $\alpha$-closed in $(X, T)$. Therefore, every fuzzy locally semi-continuous function need not be fuzzy locally $\alpha$-continuous.

**Proposition 4.4** Every fuzzy locally semi-continuous function is fuzzy locally b-continuous.

**Remark 4.3** The converse of the above Proposition 4.4 need not be true.

**Example 4.3** Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows:

$\lambda_1(a) = 0.6, \lambda_1(b) = 0.8; \lambda_2(a) = 0.5, \lambda_2(b) = 0.4; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9$. Define the fuzzy topology on $X$ as $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Define the fuzzy topology on $Y$ as $S = \{0, 1, \lambda\}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.3, \lambda(b) = 0.4$. Clearly $(X, T)$ and $(Y, S)$ are fuzzy topological spaces. Define $f: (X, T) \to (Y, S)$ as $f(a) = b, f(b) = a$. 

53
Then, \( f^{-1}(1-\lambda) = (0.6, 0.7) \) is fuzzy b-closed. Thus, for \( \lambda_3 \in T \) and for the fuzzy b-closed set \( f^{-1}(1-\lambda) \), \( \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda) \) is fuzzy locally b-closed but not fuzzy locally semi-closed in. Therefore, every fuzzy locally b-continuous function need not be fuzzy locally semi-continuous.

### Proposition 4.5
Every fuzzy locally continuous function is fuzzy locally pre-continuous.

### Remark 4.4
The converse of the above Proposition 4.5 need not be true.

### Example 4.4
Let \( X = \{ a, b \} \) and let \( \lambda_1, \lambda_2, \lambda_3 \in I^X \) be defined as follows: \( \lambda_1(a) = 0.5, \lambda_1(b) = 0.6; \lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.9, \lambda_3(b) = 0.8 \). Define the fuzzy topology on \( X \) as \( T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \} \). Define the fuzzy topology on \( Y \) as \( S = \{ 0, 1, \lambda \} \), where \( \lambda \in I^X \) is defined as \( \lambda(a) = 0.5, \lambda(b) = 0.3 \). Clearly \( (X, T) \) and \( (Y, S) \) are fuzzy topological spaces. Define \( f : (X, T) \to (Y, S) \) as \( f(a) = b, f(b) = a \).

Then, \( f^{-1}(1-\lambda) = (0.7, 0.5) \) is fuzzy pre-closed. Thus, for \( \lambda_3 \in T \) and for the fuzzy pre-closed set \( f^{-1}(1-\lambda) \), \( \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda) \) is fuzzy locally pre-closed but not fuzzy locally closed in \( (X, T) \).

Therefore, every fuzzy locally pre-continuous function need not be fuzzy locally continuous.

### Proposition 4.6
Every fuzzy locally pre-continuous function is fuzzy locally b-continuous.

### Remark 4.5
The converse of the above Proposition 4.6 need not be true.

### Example 4.5
Let \( X = \{ a, b \} \) and let \( \lambda_1, \lambda_2, \lambda_3 \in I^X \) be defined as follows: \( \lambda_1(a) = 0.3, \lambda_1(b) = 0.2; \lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.9, \lambda_3(b) = 0.9 \). Define the fuzzy topology on \( X \) as \( T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \} \). Define the fuzzy topology on \( Y \) as \( S = \{ 0, 1, \lambda \} \), where \( \lambda \in I^X \) is defined as \( \lambda(a) = 0.5, \lambda(b) = 0.5 \). Clearly \( (X, T) \) and \( (Y, S) \) are fuzzy topological spaces. Define \( f : (X, T) \to (Y, S) \) as \( f(a) = b, f(b) = a \).

Then, \( f^{-1}(1-\lambda) = (0.5, 0.5) \) is fuzzy b-closed. Thus, for \( \lambda_3 \in T \) and for the fuzzy b-closed set \( f^{-1}(1-\lambda) \), \( \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda) \) is fuzzy locally b-closed but not fuzzy locally pre-closed in \( (X, T) \).

Therefore, every fuzzy locally b-continuous function need not be fuzzy locally pre-continuous.

### Remark 4.6
Clearly the above discussions give the following implications:

---

### 5. A VIEW ON FUZZY LOCALLY b-COMPACTNESS

#### Definition 5.1
Let \( (X, T) \) be a fuzzy topological space. The collection \( \{ \lambda_i \in I^X : \lambda_i \text{ is fuzzy locally b-open}, i \in J \} \) is called the fuzzy locally b-open cover of \( (X, T) \) if \( \bigvee_{i \in J} \lambda_i = 1 \).
Definition 5.2 Let \((X, T)\) be a fuzzy topological space. The collection \(\{ \lambda_i \in I^X : \lambda_i \text{ is fuzzy locally b-closed}, i \in J \}\) is called the fuzzy locally b-closed cover of \((X, T)\) if \(\bigvee_{i \in J} \lambda_i = 1\).

Definition 5.3 A fuzzy topological space \((X, T)\) is said to be
(a) fuzzy locally b-compact if every fuzzy locally b-open cover of \((X, T)\) has a finite subcover.
(b) fuzzy locally countably b-compact if every fuzzy locally b-open countable cover of \((X, T)\) has a finite subcover.
(c) fuzzy locally b-Lindelof if every fuzzy locally b-open cover of \((X, T)\) has a countable subcover.
(d) fuzzy locally b-closed-compact if every fuzzy locally b-closed cover of \((X, T)\) has a finite subcover.
(e) fuzzy locally countably b-closed-compact if every fuzzy locally b-closed countable cover of \((X, T)\) has a finite subcover.
(f) fuzzy locally b-closed-Lindelof if every fuzzy locally b-closed cover of \((X, T)\) has a countable subcover.

Definition 5.4 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces. Any function \(f : (X, T) \to (Y, S)\) is said to be fuzzy locally b-continuous if \(f^{-1}(\rho) \in I^X\) is a fuzzy locally b-open set for every fuzzy open set \(\rho \in I^Y\).

Definition 5.5 Any fuzzy topological space \((X, T)\) is said to be a fuzzy locally b-\(T_{1/2}\) space if every fuzzy locally b-open set in \((X, T)\) is a fuzzy open set.

Proposition 5.1 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces and let \(f : (X, T) \to (Y, S)\) be a fuzzy locally b-continuous surjection. If \((X, T)\) is fuzzy locally b-compact, then \((Y, S)\) is fuzzy mildly compact.

Proposition 5.2 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces and let \(f : (X, T) \to (Y, S)\) be a fuzzy locally b-continuous surjection. If \((X, T)\) is fuzzy locally b-Lindelof, then \((Y, S)\) is fuzzy mildly Lindelof.

Proposition 5.3 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces and let \(f : (X, T) \to (Y, S)\) be a fuzzy locally b-continuous surjection. If \((X, T)\) is fuzzy locally countably b-compact, then \((Y, S)\) is fuzzy mildly countably compact.

Proposition 5.4 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces and let \(f : (X, T) \to (Y, S)\) be a fuzzy locally b-continuous surjection. If \((X, T)\) is fuzzy locally b-closed-compact. Then \((Y, S)\) is fuzzy mildly compact.

Proposition 5.5 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces and let \(f : (X, T) \to (Y, S)\) be a fuzzy locally b-continuous surjection. If \((X, T)\) is fuzzy locally b-closed-Lindelof, then \((Y, S)\) is fuzzy mildly Lindelof.

Proposition 5.6 Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces and let \(f : (X, T) \to (Y, S)\) be a fuzzy locally b-continuous surjection. If \((X, T)\) is fuzzy locally countably b-closed-compact, then \((Y, S)\) is fuzzy mildly countably compact.
A STUDY ON FUZZY LOCALLY b-CLOSED SETS

**Proposition 5.7** Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces. If \((X, T)\) is a fuzzy compact and fuzzy locally \(b\overline{T}_{1/2}\) space and \(f: (X, T) \rightarrow (Y, S)\) is a fuzzy locally \(b\)-continuous surjection, then \((Y, S)\) is also a fuzzy compact space.

**Acknowledgement:** The authors express their sincere thanks to the referees for their valuable comments regarding the improvement of the paper.

**REFERENCES**

(1) Amudhambigai B., Uma M. K. and Roja E., \(r\)-Fuzzy \(G_{b\overline{g}}\) - Locally Closed Sets and Fuzzy \(G_{b\overline{g}}\) - Locally Continuous Functions, Int. J. of Mathematical Sciences and Applications, Vol. 1, No. 3, September 2011.


(3) Benchalli S. S. and Jenifer Karnel., On Fuzzy \(b\)-open sets in Fuzzy topological space J. Computer and Mathematical Sciences 1 (2010), 127 - 134.


