

A STUDY ON FUZZY LOCALLY b-CLOSED SETS

G. VASUKI & T.PREMAKUMARI

Associate Professor of Mathematics

Dr. B.AMUDHAMBIGAI
Assistant Professor of Mathematics
Department of Mathematics
Sri Sarada College for Women, Salem-16
Tamil Nadu, India.

T.PREMAKUMARI
Sri Sarada College for Women, Salem-16
Tamil Nadu, India.

ABSTRACT

In this paper the concept of fuzzy locally b-closed sets is introduced and its inter relations with other types of locally closed sets are studied with suitable counter examples. Equivalently the inter relations of fuzzy locally b-continuous functions with other types of fuzzy locally continuous functions are discussed with necessary counter examples. Also the concepts of fuzzy locally b-compact spaces, fuzzy locally b-Lindelof spaces and fuzzy locally b-closed compact spaces are introduced and some of their charecterizations and properties are established.

Key words

fuzzy locally b-closed sets, fuzzy locally b-continuous functions, fuzzy locally b-compact spaces and fuzzy locally b-Lindelof spaces

1. INTRODUCTION AND PRELIMINARIES

1.1 Introduction

The concept of fuzzy sets was introduced by Zadeh [11] in his classical paper. Fuzzy sets have applications in many fields such as information [9] and control [10]. The first step of locally closedness was done by Bourbaki [5]. Ganster and Reilly used locally closed sets in [8] to define L_c - continuity and L_c - compactness. The concepts of r - fuzzy G_δ - \tilde{g} -locally closed sets and fuzzy G_δ - \tilde{g} -locally continuous functions were studied by Amudhambigai, Uma and Roja [1]. The concepts of fuzzy slightly β -continuity, fuzzy β -Lindelof, fuzzy mildly compact and fuzzy countably β -closed compact were introduced by Erdal Ekici [7]. In this paper the concept of fuzzy locally b-closed sets is introduced and its inter relations with other types of locally closed sets are studied with suitable counter examples. Equivalently the inter relations of fuzzy locally b-continuous functions with other types of fuzzy locally continuous functions are discussed with necessary counter examples. Also the concepts of fuzzy locally b-compact spaces, fuzzy locally b-Lindelof spaces and

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fuzzy locally b-closed compact spaces are introduced and some of their charecterizations and properties are established.

1.2 PRELIMINARIES

Definition : 1.2.2 [2,6] A fuzzy topological space (X, T) is said to be **fuzzy compact** if every fuzzy open cover of (X, T) has a finite subcover.

Definition : 1.2.4 [4] Any $\lambda \in I^X$ in a fuzzy topological space (X, T) is said to be a

- (i) **fuzzy α -closed set** (briefly, $F\alpha$ -cls) if $\lambda \geq \text{cl}(\text{int}(\text{cl}(\lambda)))$
- (ii) **fuzzy pre-closed set** (briefly, Fp -cls) if $\lambda \geq \text{cl}(\text{int}(\lambda))$.
- (iii) **fuzzy semi-closed set** (briefly, Fs -cls) if $\lambda \geq \text{int}(\text{cl}(\lambda))$.

Definition : 1.2.7 [3] Any fuzzy set $\lambda \in I^X$ in a fuzzy topological space (X, T) is said to be a **fuzzy b-closed set** (briefly, Fb -cls) if $\lambda \geq \text{cl}(\text{int}(\lambda)) \wedge \text{int}(\text{cl}(\lambda))$.

Definition : 1.2.8 [6] Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f: (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy continuous function** if $f^{-1}(\lambda)$ is fuzzy open in (X, T) for each fuzzy open set λ in (Y, S) .

Definition : 1.2.9 [7] A fuzzy topological space (X, T) is said to be **fuzzy β -compact** if every fuzzy β -open cover of (X, T) has a finite subcover.

(a) **fuzzy countably β -compact** if every fuzzy β -open countably cover of (X, T) has a finite subcover.

(b) **fuzzy β -Lindelof** if every fuzzy β -open cover of (X, T) has a countable subcover.

(c) **fuzzy mildly compact** if every fuzzy clopen cover of (X, T) has a finite subcover.

(d) **fuzzy mildly countably compact** if every fuzzy clopen countably cover of (X, T) has a finite subcover.

(e) **fuzzy mildly Lindelof** if every fuzzy clopen cover of (X, T) has a countable subcover.

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Definition 2.1 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally closed set** (briefly, **FLcls**) if $\lambda = \mu \wedge \gamma$, where μ is a fuzzy open set and γ is a fuzzy closed set. Its complement is called a **fuzzy locally open set**.

Definition 2.2 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally α -closed set** (briefly, **FL α -cls**) if $\lambda = \mu \wedge \gamma$, where μ is a fuzzy open set and γ is a fuzzy α -closed set. Its complement is called a **fuzzy locally α -open set**.

Definition 2.3 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally semi-closed set** (briefly, **FLs-cls**) if $\lambda = \mu \wedge \gamma$, where μ is a fuzzy open set and γ is a fuzzy semi-closed set. Its complement is called a **fuzzy locally semi-open set**.

Definition 2.4 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally pre-closed set** (briefly, **FLp-cls**) if $\lambda = \mu \wedge \gamma$, where μ is a fuzzy open set and γ is a fuzzy pre-closed set. Its complement is called a **fuzzy locally pre-open set**.

Definition 2.5 Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy locally b-closed set** (briefly, **FLb-cls**) if $\lambda = \mu \wedge \gamma$, where μ is a fuzzy open set and γ is a fuzzy b-closed set. Its complement is called a **fuzzy locally b-open set**.

Proposition 2.1 Every fuzzy locally closed set is fuzzy locally α -closed.

Remark 2.1 The converse of the above Proposition 2.1 need not be true.

Example 2.1 Let $X = \{a, b\}$ and $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.5, \lambda_1(b) = 0.4; \lambda_2(a) = 0.5, \lambda_2(b) = 0.2; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9$. Define the fuzzy topology $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.5, \gamma(b) = 0.7$. Then γ is fuzzy α -closed. Thus, for $\lambda_3 \in T, \lambda_3 \wedge \gamma = (0.5, 0.7) = \lambda$ is **fuzzy locally α -closed**. But, λ is not a fuzzy locally closed set,

Proposition 2.2 Every fuzzy locally α -closed set is fuzzy locally semi-closed.

Remark 2.2 The converse of the above Proposition 2.2 need not be true.

Example 2.2 Let $X = \{a, b\}$ and $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.3, \lambda_1(b) = 0.4; \lambda_2(a) = 0.4, \lambda_2(b) = 0.4; \lambda_3(a) = 0.8, \lambda_3(b) = 0.9$. Define the fuzzy topology $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.6, \gamma(b) = 0.5$. Then γ is fuzzy semi-closed. Thus, for $\lambda_3 \in T, \lambda_3 \wedge \gamma = (0.6, 0.5) = \lambda$ is **fuzzy locally semi-closed**. But, λ is not a fuzzy locally α -closed set.

Proposition 2.3 Every fuzzy locally semi-closed set is fuzzy locally b-closed.

Remark 2.3 The converse of the above Proposition 2.3 need not be true.

Example 2.3 Let $X = \{a, b\}$ and $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.6, \lambda_1(b) = 0.8; \lambda_2(a) = 0.5, \lambda_2(b) = 0.4; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9$. Define the fuzzy topology $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.6, \gamma(b) = 0.7$. Then γ is fuzzy b-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy b-closed set $\gamma, \lambda_3 \wedge \gamma = (0.6, 0.7) = \lambda$ is **fuzzy locally b-closed**. But, λ is not a fuzzy locally semi-closed set.

Proposition 2.4 Every fuzzy locally closed set is fuzzy locally pre-closed.

Remark 2.4 The converse of the above Proposition 2.4 need not be true.

Example 2.4 Let $X = \{a, b\}$ and $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.5, \lambda_1(b) = 0.6; \lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.9, \lambda_3(b) = 0.8$. Define the fuzzy topology $T = \{0, 1, \lambda_1, \lambda_2, \lambda_3\}$. Clearly (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.7, \gamma(b) = 0.5$. Then γ is fuzzy pre-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy pre-closed set $\gamma, \lambda_3 \wedge \gamma = (0.7, 0.5) = \lambda$ is **fuzzy locally pre-closed**. But, λ is not a fuzzy locally closed set.

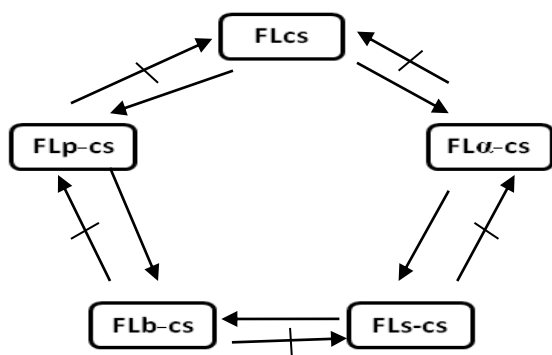
Proposition 2.5 Every fuzzy locally pre-closed set is fuzzy locally b-closed.

Remark 2.5 The converse of the above Proposition 2.5 need not be true.

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Example 2.5 Let $X = \{ a, b \}$ and $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows: $\lambda_1(a) = 0.3, \lambda_1(b) = 0.2; \lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.5, \lambda_3(b) = 0.9$. Define the fuzzy topology $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Clearly (X, T) is a fuzzy topological space. Let $\gamma \in I^X$ be defined as $\gamma(a) = 0.7, \gamma(b) = 0.5$. Then γ is fuzzy b-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy b-closed set $\gamma, \lambda_3 \wedge \gamma = (0.5, 0.5) = \lambda$ is **fuzzy locally b-closed**. But, λ is not a fuzzy locally pre-closed set.

Remark 2.6 Clearly the above discussions give the following implications :



4. A STUDY ON FUZZY LOCALLY b-CONTINUOUS FUNCTION

Definition 4.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally continuous function** (briefly, **FLcf**) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally closed and for each fuzzy closed set $\lambda \in I^Y$.

Definition 4.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally α -continuous function** (briefly, **FL α -cf**) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally α -closed for each fuzzy closed set $\lambda \in I^Y$.

Definition 4.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally semi-continuous function** (briefly, **FLs-cf**) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally semi-closed and for each fuzzy closed set $\lambda \in I^Y$.

Definition 4.4 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally pre-continuous function** (briefly, **FLp-cf**) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally pre-closed and for each fuzzy closed set $\lambda \in I^Y$.

Definition 4.5 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be a **fuzzy locally b-continuous function** (briefly, **FLb-cf**) if $f^{-1}(\lambda) \in I^X$ is fuzzy locally b-closed and for each fuzzy closed set $\lambda \in I^Y$.

Definition 4.6 Let (X, T) be a fuzzy topological space. For a fuzzy set λ of X , the **fuzzy locally α -closure** (briefly, **FL α -cl**) and **fuzzy locally α -interior** (briefly, **FL α -int**) of λ are defined respectively, as $FL\alpha-cl(\lambda) = \bigwedge \{ \mu \in I^X : \mu \geq \lambda; \mu \text{ is fuzzy locally } \alpha\text{-closed set} \}$ and

$FL\alpha\text{-int}(\lambda) = \vee \{ \mu \in I^X : \mu \leq \lambda; \mu \text{ is fuzzy locally } \alpha\text{-open set} \}$.

Proposition 4.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Then for any function $f: (X, T) \rightarrow (Y, S)$ the following statements are equivalent :

- (a) f is fuzzy locally α -continuous.
- (b) For every $\lambda \in I^X, f(FL\alpha\text{-cl}(\lambda)) \leq cl(f(\lambda))$.
- (c) For every $\lambda \in I^Y, f^{-1}(cl(\lambda)) \geq FL\alpha\text{-cl}(f^{-1}(\lambda))$.
- (d) For every $\lambda \in I^Y, f^{-1}(int(\lambda)) \leq FL\alpha\text{-int}(f^{-1}(\lambda))$.

Proposition 4.2 Every fuzzy locally continuous function is fuzzy locally α -continuous.

Remark 4.1 The converse of the above Proposition 4.2 need not be true.

Example 4.1 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.5, \lambda_1(b) = 0.4; \lambda_2(a) = 0.5, \lambda_2(b) = 0.2; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9$. Define the fuzzy topology on X as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Define the fuzzy topology on Y as $S = \{ 0, 1, \lambda \}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.3, \lambda(b) = 0.5$. Clearly (X, T) and (Y, S) are fuzzy topological spaces. Define $f: (X, T) \rightarrow (Y, S)$ as $f(a) = b, f(b) = a$. Then, $f^{-1}(1-\lambda) = (0.5, 0.7)$ is fuzzy α -closed. Thus, for $\lambda_3 \in T$ and for the fuzzy α -closed set $f^{-1}(1-\lambda), \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally α -closed but not fuzzy locally closed in (X, T) .

Therefore, every fuzzy locally α -continuous function need not be fuzzy locally continuous.

Proposition 4.3 Every fuzzy locally α -continuous function is fuzzy locally semi-continuous.

Remark 4.2 The converse of the above Proposition 4.3 need not be true.

Example 4.2 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.3, \lambda_1(b) = 0.4; \lambda_2(a) = 0.4, \lambda_2(b) = 0.4; \lambda_3(a) = 0.8, \lambda_3(b) = 0.9$. Define the fuzzy topology on X as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Define the fuzzy topology on Y as $S = \{ 0, 1, \lambda \}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.5, \lambda(b) = 0.4$. Clearly (X, T) and (Y, S) are fuzzy topological spaces. Define $f: (X, T) \rightarrow (Y, S)$ as $f(a) = b, f(b) = a$. Then, $f^{-1}(1-\lambda) = (0.6, 0.5)$ is fuzzy semi-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy semi-closed set $f^{-1}(1-\lambda), \lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally semi-closed but not fuzzy locally α -closed in (X, T) . Therefore, every fuzzy locally semi-continuous function need not be fuzzy locally α -continuous.

Proposition 4.4 Every fuzzy locally semi-continuous function is fuzzy locally b-continuous.

Remark 4.3 The converse of the above Proposition 4.4 need not be true.

Example 4.3 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.6, \lambda_1(b) = 0.8; \lambda_2(a) = 0.5, \lambda_2(b) = 0.4; \lambda_3(a) = 0.7, \lambda_3(b) = 0.9$. Define the fuzzy topology on X as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Define the fuzzy topology on Y as $S = \{ 0, 1, \lambda \}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.3, \lambda(b) = 0.4$. Clearly (X, T) and (Y, S) are fuzzy topological spaces. Define $f: (X, T) \rightarrow (Y, S)$ as $f(a) = b, f(b) = a$.

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Then, $f^{-1}(1-\lambda) = (0.6, 0.7)$ is fuzzy b-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy b-closed set $f^{-1}(1-\lambda)$, $\lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally b-closed but not fuzzy locally semi-closed in. Therefore, **every fuzzy locally b-continuous function need not be fuzzy locally semi-continuous.**

Proposition 4.5 Every fuzzy locally continuous function is fuzzy locally pre-continuous.

Remark 4.4 The converse of the above Proposition 4.5 need not be true.

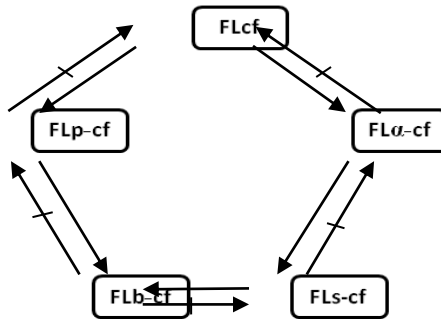
Example 4.4 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.5, \lambda_1(b) = 0.6;$
 $\lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.9, \lambda_3(b) = 0.8.$ Define the fuzzy topology on X as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Define the fuzzy topology on Y as $S = \{ 0, 1, \lambda \}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.5, \lambda(b) = 0.3.$ Clearly (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b, f(b) = a.$ Then, $f^{-1}(1-\lambda) = (0.7, 0.5)$ is fuzzy pre-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy pre-closed set $f^{-1}(1-\lambda)$, $\lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally pre-closed but not fuzzy locally closed in (X, T) . Therefore, **every fuzzy locally pre-continuous function need not be fuzzy locally continuous.**

Proposition 4.6 Every fuzzy locally pre-continuous function is fuzzy locally b-continuous.

Remark 4.5 The converse of the above Proposition 4.6 need not be true.

Example 4.5 Let $X = \{ a, b \}$ and let $\lambda_1, \lambda_2, \lambda_3 \in I^X$ be defined as follows : $\lambda_1(a) = 0.3, \lambda_1(b) = 0.2;$
 $\lambda_2(a) = 0.4, \lambda_2(b) = 0.3; \lambda_3(a) = 0.5, \lambda_3(b) = 0.9.$ Define the fuzzy topology on X as $T = \{ 0, 1, \lambda_1, \lambda_2, \lambda_3 \}$. Define the fuzzy topology on Y as $S = \{ 0, 1, \lambda \}$, where $\lambda \in I^Y$ is defined as $\lambda(a) = 0.5, \lambda(b) = 0.5.$ Clearly (X, T) and (Y, S) are fuzzy topological spaces. Define $f : (X, T) \rightarrow (Y, S)$ as $f(a) = b, f(b) = a.$ Then, $f^{-1}(1-\lambda) = (0.5, 0.5)$ is fuzzy b-closed. Thus, for $\lambda_3 \in T$ and for the fuzzy b-closed set $f^{-1}(1-\lambda)$, $\lambda_3 \wedge f^{-1}(1-\lambda) = f^{-1}(1-\lambda)$ is fuzzy locally b-closed but not fuzzy locally pre closed in (X, T) . Therefore, **every fuzzy locally b-continuous function need not be fuzzy locally pre continuous.**

Remark 4.6 Clearly the above discussions give the following implications :



5. A VIEW ON FUZZY LOCALLY b-COMPACTNESS

Definition 5.1 Let (X, T) be a fuzzy topological space. The collection $\{ \lambda_i \in I^X : \lambda_i \text{ is fuzzy locally b-open, } i \in J \}$ is called the fuzzy locally b-open cover of (X, T) if $\bigvee_{i \in J} \lambda_i = 1.$

Definition 5.2 Let (X, T) be a fuzzy topological space. The collection $\{\lambda_i \in I^X : \lambda_i \text{ is fuzzy locally b-closed, } i \in J\}$ is called the fuzzy locally b-closed cover of (X, T) if $\bigvee_{i \in J} \lambda_i = 1$.

Definition 5.3 A fuzzy topological space (X, T) is said to be

- (a) fuzzy locally b-compact if every fuzzy locally b-open cover of (X, T) has a finite subcover.
- (b) fuzzy locally countably b-compact if every fuzzy locally b-open countable cover of (X, T) has a finite subcover.
- (c) fuzzy locally b-Lindelof if every fuzzy locally b-open cover of (X, T) has a countable subcover.
- (d) fuzzy locally b-closed-compact if every fuzzy locally b-closed cover of (X, T) has a finite subcover.
- (e) fuzzy locally countably b-closed-compact if every fuzzy locally b-closed countable cover of (X, T) has a finite subcover.
- (f) **fuzzy locally b-closed-Lindelof** if every fuzzy locally b-closed cover of (X, T) has a countable subcover.

Definition 5.4 Let (X, T) and (Y, S) be any two fuzzy topological spaces. Any function $f : (X, T) \rightarrow (Y, S)$ is said to be **fuzzy locally b-continuous** if $f^{-1}(\rho) \in I^X$ is a fuzzy locally b-open set for every fuzzy open set $\rho \in I^Y$.

Definition 5.5 Any fuzzy topological space (X, T) is said to be a **fuzzy locally b- $T_{1/2}$ space** if every fuzzy locally b-open set in (X, T) is a fuzzy open set.

Proposition 5.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $f : (X, T) \rightarrow (Y, S)$ be a fuzzy locally b-continuous surjection. If (X, T) is fuzzy locally b-compact, then (Y, S) is fuzzy mildly compact.

Proposition 5.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $f : (X, T) \rightarrow (Y, S)$ be a fuzzy locally b-continuous surjection. If (X, T) is fuzzy locally b-Lindelof, then (Y, S) is fuzzy mildly Lindelof.

Proposition 5.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $f : (X, T) \rightarrow (Y, S)$ be a fuzzy locally b-continuous surjection. If (X, T) is fuzzy locally countably b-compact, then (Y, S) is fuzzy mildly countably compact.

Proposition 5.4 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $f : (X, T) \rightarrow (Y, S)$ be a fuzzy locally b-continuous surjection. If (X, T) is fuzzy locally b-closed-compact. Then (Y, S) is fuzzy mildly compact.

Proposition 5.5 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $f : (X, T) \rightarrow (Y, S)$ be a fuzzy locally b-continuous surjection. If (X, T) is fuzzy locally b-closed-Lindelof, then (Y, S) is fuzzy mildly Lindelof.

Proposition 5.6 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let $f : (X, T) \rightarrow (Y, S)$ be a fuzzy locally b-continuous surjection. If (X, T) is fuzzy locally countably b-closed-compact, then (Y, S) is fuzzy mildly countably compact.

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Proposition 5.7 Let (X, T) and (Y, S) be any two fuzzy topological spaces. If (X, T) is a fuzzy compact and fuzzy locally $b-T_{1/2}$ space and $f: (X, T) \rightarrow (Y, S)$ is a fuzzy locally b -continuous surjection, then (Y, S) is also a fuzzy compact space.

Acknowledgement: The authors express their sincere thanks to the referees for their valuable comments regarding the improvement of the paper.

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