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A Study of $f_{\lambda,\mu}(2\upsilon+3,1)$-structure in Tangent Bundle

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Abstract

The idea of f-structure manifold on a differentiable manifold was initiated and developed by Yano [1], Ishihara and Yano [2], Goldberg [3] and amongothers. The horizontal and complete lifts from a differentiable manifold M^n of class C to its cotangent bundles have been studied by Yano and Patterson [4,5]. Yano andIshihara [6] have studied lifts of an .f-structure in the tangent and cotangent bundles. The purpose of this paper is to obtain integrability conditions of astructure satisfying $f_{\lambda,\mu}(2\upsilon+3,1)$ -structure in the tangent bundle.

Keywords : Tangent bundle, Complete lift, F-structure, Integrability, Distributions.

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1. Introduction:

Let M^n be an n-dimensional differentiable manifold of class C^∞. Suppose there exists on M^n a non-null tensor field f of type (1,1) of class C^∞ and of rank r satisfying

$$(f^{2\upsilon+3}+\lambda^2 f)(f^{2\upsilon+3}+\mu^2 f)=0 \quad , \lambda\neq\mu \tag{1.1}$$

Let us defined on such M^n tensor fields l and m of type (1,1) as follows

$$l=\frac{f^{2\upsilon+2}+\lambda^2}{\lambda^2-\mu^2} \quad , \quad m=\frac{f^{2\upsilon+2}+\lambda^2}{\mu^2-\lambda^2} \tag{1.2}$$

Then it can be easily shown that

$$l^2=l \quad , \quad m^2=m \quad , \quad lm=ml=0 \quad , \quad l+m=1 \tag{1.3}$$

Thus the operators l and m when applied to tangent space of M^n at a point all complementary projection operators. Thus there exist complementary distribution L and M corresponding to projection operators l and m respectively. Let us call such structure as $f_{\lambda,\mu}(2\upsilon+3,1)$ -structure**.**

For the manifold M^n equipped with $f_{\lambda,\mu}(2\upsilon+3,1)$ -structure, the following result can be proved easily

$$\begin{aligned}
&\text{(i)} \quad fl=\frac{f^{2\upsilon+3}+f\lambda^2}{\lambda^2-\mu^2} \quad , \quad fm=\frac{f^{2\upsilon+3}+f\lambda^2}{\mu^2-\lambda^2}\\
&\text{(ii)} \quad f^2 l=-\mu^2 l \quad , \quad f^2 m=-\lambda^2 m\\
&\text{(iii)} \quad m-l=\frac{2f^{2\upsilon+2}+\mu^2+\lambda^2}{\mu^2-\lambda^2}
\end{aligned} \tag{1.4}$$

2. Complete Lift on $f_{\lambda,\mu}(2\upsilon+3,1)$-structure in Tangent Bundle

Let M be an n-dimensional differentiable manifold of class C^{∞} and $T_p(M^n)$ the tangent space at a point p of M^n and $T(M^n) = \bigcup_{p\varepsilon M^n} T_p(M^n)$ is the tangent bundle over the manifold M^n.

Let us denote by $T_s^r(M^n)$, the set all tensor fields of class C^{∞} and of type (r,s) in M^n and $T(M^n)$ be the tangent bundle over M^n .the complete lift F^C of an element of $T_1^1(M^n)$ with local components F_i^h has components of the form [5]

$$F^C : \begin{bmatrix} F_i^h & 0 \\ \delta_i^h & F_i^h \end{bmatrix} \tag{2.1}$$

Now we obtain the following results on the complete lift of F satisfying (1.1).

Theorem (2.1): For $F \varepsilon T_1^1(M^n)$, the complete lift F^C of F is $f_{\lambda,\mu}(2\upsilon+3,1)$-structure iff it is for F is of rank r iff F^C is of rank $2r$.

Proof: Let F, $G \varepsilon T_1^1(M^n)$ Then we have [5]

$$(FG)^C = F^C G^C \tag{2.2}$$

replacing G by F in (2.2) we obtain

$$(FF)^C = F^C F^C$$

or

$$(F^2)^C = (F^C)^2 \tag{2.3}$$

Now putting $G = F^{2\upsilon+2}$ in (2.2) since G is (1,1) tensor field therefore $F^{2\upsilon+2}$ is also (1,1) tensor field so we obtain

$$(FF^{2\upsilon+2})^C = F^C (F^{2\upsilon+2})^C$$

which is in view of (2.3) becomes

$$(FF^{2\upsilon+2})^C = F^C (F^{2\upsilon+2})^C \tag{2.4}$$

Taking complete lift on both sides of equation (1.1) we get

$$(f^{2\upsilon+3} + \lambda^2 f)^C (f^{2\upsilon+3} + \mu^2 f)^C = 0$$

$$[(f^{2\upsilon+3})^C + \lambda^2 f^C][(f^{2\upsilon+3})^C + \mu^2 f^C] = 0$$

$$[(f^C)^{2\upsilon+2} + \lambda^2 f^C][(f^C)^{2\upsilon+2} + \mu^2 f^C] = 0 \tag{2.5}$$

thus equation (1.1) and (2.5) are equivalent. The second part of the theorem follows in view of equation (2.1).

Let F satisfying (1.1) be an F -structure of rank r in M^n .Then the complete lifts l^C of l

and m^C of m are complementary projection tensors in $T_p(M^n)$ .Yhus there exist in

$T\ (M^n)$ .Thus there exist in $T\ (M^n)$ two complementary distributions L^C and M^C

determined by l^C and m^C respectively.

Theorem (2.2): The complete lift of M^C of the distribution M in $T\ (M^n)$ is integrable iff M is integrable in M^n .

Proof: It is known that the distribution M is integrable in M^n iff [2]

$$l(mX, mY) = 0 \text{ , for any } X, Y\varepsilon T_0^1(M^n) \tag{2.6}$$

Taking complete lift on both sides of equations (2.6) we get

$$l^C(m^C X^C, m^C Y^C) = 0 \tag{2.7}$$

where $l^C = (I-m)^C = I - m^C$, is the projection tensor complementary of m^C . Thus the conditions (2.6) and (2.7) are equivalent.

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