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Strongly Magic Squaresand Group Morphisms

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Abstract

A magic square is a square array of numbers where the rows, columns, diagonals and co-diagonals add up to the same number. Magic squares have been known in India from very early times. The renownedmathematician Ramanujan had immense contributions in the field of Magic Squares. The paper discuss about a well-known class of magic squares; the strongly magic square.The strongly magic square is a magic squarewith a stronger property that the sum of the entries of the sub-squares taken without any gaps between the rows or columns is also the magic constant. In this papera generic definition for Strongly Magic Squares is given. Afunction on strongly magic squares is also defined and it is proved to be a group homomorphism and isomorphism. The paper also sheds light on some applications of magic squares.

Keywords: - Magic Square, Magic Constant, Strongly Magic Square (SMS), Homomorphism, Isomorphism, Data hiding, Water Marking Scheme,Electrostatic potential
2010 Mathematics Subject Classification Code:- 20K30,20L05

I. INTRODUCTION

Magic squares date back in the first millennium B.C.E in China [1], developed in India and Islamic World in the firstmillennium C.E, and found its way to Europe in the later Middle Ages [2]and to sub-Saharan Africa not much after [3].Magic squares generally fall into the realm of recreational mathematics [4, 5], however a few times in the past century and more recently, they have become the interest of more-serious mathematicians.SrinivasaRamanujanhad contributed a lot in the field of magic squares. Ramanujan's work on magic squares is presented in detail inRamanujan's Notebooks [6].A normal magic square is a square array of consecutive numbers from $1 \dots n^2$ where the rows, columns, diagonals and co-diagonals add up to the same number. The constant sum is called magic constant or magic number. Along with the conditions of normal magic squares, strongly magic square of order 4 have a stronger property that the sum of the entries of the sub-squares taken without any gaps between the rows or columns is also the magic constant [7]. There are many recreational aspects of strongly magic squares. But, apart from the usual recreational aspects, it is found that these strongly magic squares possess advanced mathematical properties.

II. NOTATIONS AND MATHEMATICAL PRELIMINARIES

(A) Magic Square

A magic square of order n is an n^{th} order matrix $[a_{ij}]$ such that

$$\sum_{j=1}^{n} a_{ij} = \rho for i = 1,2, \ldots .. n \ldots \ldots \ldots . (1)$$

$$\sum_{j=1}^{n} a_{ji} = \rho for i = 1,2, \ldots .. n \ldots \ldots \ldots . (2)$$

$$\sum_{i=1}^{n} a_{ii} = \rho \ , \ \sum_{i=1}^{n} a_{i,n-i+1} = \rho \ldots \ldots \ldots . (3)$$

Equation (1) represents the row sum, equation (2) represents the column sum, equation (3) represents the diagonal and co-diagonal sum and symbol ρ represents the magic constant. [8]

(B) Magic Constant

The constant ρ in the above definition is known as the magic constant or magic number. The magic constant of the magic square A is denoted as $\rho(A)$.

(C) Strongly magic square (SMS): Generic Definition

Let $A = [a_{ij}]$ be a matrix of order $n^2 \times n^2$, such that

$$\sum_{j=1}^{n^2} a_{ij} = \rho \ for \ i = 1,2, \ldots .. n^2 \qquad \ldots \ldots \ldots \ldots .. \ldots . (4)$$

$$\sum_{j=1}^{n^2} a_{ji} = \rho \ for \ i = 1,2, \ldots .. n^2 \qquad \ldots \ldots \ldots \ldots \ldots \ldots ... (5)$$

$$\sum_{i=1}^{n^2} a_{ii} = \rho \ , \qquad \sum_{i=1}^{n^2} a_{i,n^2-i+1} = \rho \qquad \ldots \ldots \ldots . (6)$$

$$\sum_{l=0}^{n-1} \sum_{k=0}^{n-1} a_{i+k,j+l} = \rho \ for \ i,j = 1,2, \ldots .. n^2 \qquad \ldots \ldots \ldots . \ldots . (7)$$

$$where\ the\ subscripts\ are\ congruent\ modulo\ n^2$$

Equation (4) represents the row sum, equation (5) represents the column sum, equation (6) represents the diagonal & co-diagonal sum, equation (7) represents the $n \times n$ sub-square sum with no gaps in between the elements of rows or columns and is denoted as ${M_{0C}}^{(n)} or {M_{0R}}^{(n)}$ and ρ is the magic constant.

Note: The n^{th} order subsquare sum with k column gaps or k row gaps is generally denoted as $M_{kC}{}^{(n)}$ or $M_{kR}{}^{(n)}$respectively.

(D) Group homomorphism

A mapping ϕ from a group $< G, * >$into a group $< G', *' >$ is a homomorphism of G into G' if

$\phi(a * b) = \phi(a) *' \phi(b)$for all $a, b \in$ G [9]

(E) Group isomorphism

A one to one onto homomorphism ϕ from a group $< G, * >$into a group $< G', *' >$ is defined as isomorphism [9]

(F) A one to one and onto mapping

A function ϕ :X → Yis one to one if $\phi(x_1) = \phi(x_2)$ only when $x_1 = x_2$.

The function ϕ is onto of Y if the range of ϕ is Y. [9]

(G) Other Notations

1. R denotes the set of all real numbers.
2. SM_sdenote the set of all strongly magic squares of order $n^2 \times n^2$
3. $SM_{S(a)}$ denote the set of all strongly magic squares of the form $[a_{ij}]_{n^2 \times n^2}$ such that$a_{ij} = a$ for every $i, j = 1,2, \dots n^2$. Here A is denoted as $[a]$, i.e. If $A \in SM_{S(a)}$ then $\rho(A) = n^2 a$

III. PROPOSITIONS AND THEOREMS

Proposition 1

If A and B are two Strongly magic squares of order $n^2 \times n^2$ with $\rho(A) = a$and $\rho(B) = b$, then $C = (\lambda + \mu)(A + B)$ is also a Strongly magic square with magic constant $(\lambda + \mu)\big(\rho(A) + \rho(B)\big)$;for every $\lambda, \mu \in R$

Proof:

Let $A = [a_{ij}]_{n^2 \times n^2}$ and $B = [b_{ij}]_{n^2 \times n^2}$

Then$C = (\lambda + \mu)(A + B)$

$= [(\lambda + \mu)(a_{ij} + b_{ij})]$

Sum of the ith row elements of

$$C = \sum_{j=1}^{n^2} c_{ij} = (\lambda + \mu)\left(\sum_{j=1}^{n^2}(a_{ij}) + \sum_{j=1}^{n^2}(b_{ij})\right)$$

$$= (\lambda + \mu)(a + b)$$

$$= (\lambda + \mu)(\rho(A) + \rho(B))$$

A similar computation holds for column sum,diagonals sum and sum of the $n \times n$ sub squares

From the above propositions the following results can be obtained by putting suitable values for λ, and μ

Results

If for every $\lambda, \mu \in R$ and $A , B \in SM_s$,

1.1) $\lambda\,(A+B) \in SM_s$ $with$ $\rho\,(\lambda\,(A+B)) = \lambda\,(\rho(A) + \rho(B))$

1.2) $(A+B) \in SM_s$ $with$ $\rho(A+B) = \rho(A) + \rho(B)$

Proposition 2

The mapping $\phi : S_{M_s} \rightarrow R$defined by $\phi\,(A) = \rho(A)$, $\forall A \in S_{M_s}$is a group homomorphism.

Proof

Let $A , B \in S_{M_s}$,then $\phi\,(A+B) = \rho(A+B) = \rho(A) + \rho(B)$ (By Result 1.2)

$= \phi\,(A) + \phi\,(B)$

Proposition 3

The mapping $\phi : SM_{S(a)} \rightarrow R$defined by $\phi\,(A) = \rho(A)$, $\forall A \in SM_{S(a)}$is a group homomorphism

Proof

It can be easily verified since $SM_{S(a)} \subset S_{M_s}$

Theorem 4

The mapping $\phi : SM_{S(a)} \rightarrow R$defined by $\phi\,(A) = \rho(A)$, $\forall A \in SM_{S(a)}$is a group isomorphism.

Proof

Let $A, B \in SM_{S(a)}$ then $A = [a]$, $B = [b]$ such that $\rho(A) = n^2 a$ and $\rho(B) = n^2 b$

i. To show that ϕ is one to one

$\phi\,(A) = \phi\,(B) \Rightarrow \rho(A) = \rho(B) \Rightarrow n^2 a = n^2 b$

$\Rightarrow a = b$

ii. To show that ϕ is onto

For every $a \in R$, there exists $A = \left[\frac{a}{n^2}\right] \in SM_{S(a)}$such that $\rho(A) = a.$

Since ϕ is 1-1 and onto and from Proposition 3, it can be deduced.

IV. APPLICATIONS OF MAGIC SQUARE

A. Data Hiding

Data hiding scheme using magic squares can be used to efficiently hide the data at the LSBs of a host image. First, data is exchanged by the transposition square which is reordered by a sequential of magic squares. The secret data is replaced by matching, encoding and substituting from a constructed table. The hidden data is embedded into the LSBs of the host images. The stego images with embedded data are imperceptible. The needed parameters formed a private key are encrypted and sent to the receiver in the Internet. Moreover, if any one of them is incorrect, it is very difficult to find the correct texts. [10]

B. Electrostatic Potential

Some of contour plots of natural magic squares electrostatic potential are symmetrical There might be a connection with sets of singular values of magic squares (Fig.1).

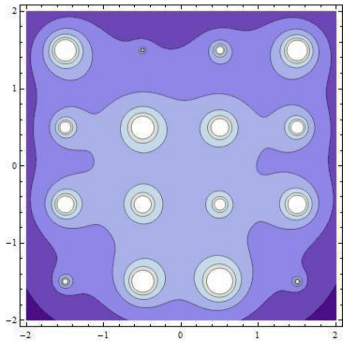

Fig1: Contour plot of electro-static potential of a sample 4×4 natural magic square

The electrostatic potential at the center of all associative magic squares of an arbitrary order and also for natural magic squares of order 3 and 4 is constant.

$$\varphi = \frac{1}{4\pi\varepsilon_0}(n^2 + 1) \times C$$

Moreover, electrostatic potential at the center of associative magic squares is equal to average of minimum and maximum potential at the center of normal squares. [12].

C. Watermarking Scheme

To protect sensitive multimedia data from being illegally modified water marking scheme is used. Studies propose a fragile watermarking scheme to detect illegitimate alterations of a watermarked image. There are studies that propose a method that hides a magic square into a gray scale image in a block-by-block fashion by using the least significant bit replacement. [11]

V. CONCLUSION

While magic squares are recreational in grade school, they may be treated somewhat more seriously in different mathematical courses. The study of strongly magic squares is an emerging innovative area in which mathematical analysis can be done. Here some advanced properties regarding strongly magic squares are described. This paper touches upon some interesting applications of magic squares, including data hiding, watermarking and electrostatic potential. Physical application of magic squares is still a new topic that needs to be explored more. There are many interesting ideas for research in this field.

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