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SPATIOTEMPORAL ANALYSIS OF A PREY PREDATOR MODEL WITH TROPHIC INTERACTION WITH HARVESTING

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Abstract. In this paper, we have considered a biological economic model based on prey-predator dynamics where prey and predator species continuously harvested and predation is considered with Holling type -II functional response. The dynamic behavior of the proposed biological economic prey predator model is discussed. We assumed that both prey and predator species follows logistic growth. The local and global stability analysis has been specified using Routh -Hurwitz criteria and by constructing Lyapunov function respectively. Biological and Bionomical equilibriums of the system are derived. Mathematical formulation of the optimal harvesting policy is given and its solution is derived in the equilibrium case by using Pontryagin's maximum principle.spatiotemporal changing aspects of the proposed model are also studied. It is also provided the computer simulations for verifying the results.

1. Introduction

Biomathematics is an interdisciplinary subject with a vast and exponentially growing literature pertaining to different disciplines. A large number of mathematical models have been developed to get an insight into complex biological, ecological and physiological situations. A variety of Mathematical techniques have been employed to solve these models. These include techniques for the solution of differential, difference, integral delay differential and integro differential equations as

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well as useful techniques of linear, non-linear, dynamic, stochastic programming, calculus of variations, maximum principle and so on. Some of the latest results of algebraic topology, fuzzy sets and catastrophe theory have been successfully employed to probe deeply into the problems of life sciences. Most of the biological systems are essentially based on systems of Non-linear ordinary differential equations. Both mathematical modeling and simulation are very important in recent studies of mathematical Biology. The development of commonly used resources like forestry, fishery, and wildlife is linked with the applications of mathematical biology. Now a day's many authors are emphasizing on the interaction between biology and mathematics. The prey-predator system is an important population model and has been studied by many authors [26, 38, 44, 45]. It is assumed in the classical predator-prey model that each individual predator possesses the same ability to attack prey In recent years, the optimal management of renewable resources, which has a direct relationship to sustainable development, has been studied extensively by K.S.Chaudhuri [5], T.K.Kar et.al[26] and W.Wang, L.Chen [43], Clark [6]. At present people are facing the problems due to shortage of resources. Extensive and unregulated harvesting of marine fishes can even lead to the extinction of several fish species. This problem can be addressed by arranging marine reserved zones, where fishing and other activities are prohibited. This marine reserve not only protects species inside the reserve area but also increase fish abundance in adjacent areas. The model of ecological system reflecting these problems has been given by T.K. Kar et.al [26] and Rui Zhang et.al [45]. Wendy-Wang et.al [44], considered prey-predator model with a stage structure in which predators are split in to immature predators and matured predators. They also assumed that the matured predators catch the prey and provide food for the immature predators. Rui Zhang et.al [45] considered a prey predator fishery model with prey dispersed in two patch environment, one is free zone for fishing and other is reserved zone where fishing is prohibited. A.K.Sarkar [37] considered a mathematical model of an out breaks that links the trophic structure of primary and secondary producer in the estuary. They also discussed about the results that are qualitatively resemble with those observed in the estuary and thereby offers an insight for the factors that sustain a bloom. In this paper our aim is to study the existence of equilibriums, local, global stabilities of a biological economic prey-predator model by assuming the growth of both prey and predator logistically. We also derived the inequalities of bionomic equilibrium and the mathematical formulation of optimal harvesting policy for the prey-predator model with Holling type-II.

2. Mathematical formulation

We consider the following dynamical system as an ecological model of a prey-predator Holling type-II interaction. This also includes the harvesting efforts for both prey and predator species. The inclusion of logistic growth in prey and as well as in predator is considered and the corresponding mathematical equations are as follows.

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{axy}{b+x} - q_1E_1x \tag{2.1}$$

$$\frac{dy}{dt} = \frac{axy}{b+x} + dy\left(1 - \frac{x}{k}\right) - cy^2 - q_2E_2y \tag{2.2}$$

where $x(t)$ is the biomass density of the prey species and $y(t)$ is the biomass density of predator species at time t in any patchy aquatic environment with the initial conditions $r - q_1E_1 > 0$, $d - q_2E_2 > 0$ where r represents intrinsic growth rate of prey species, k represents carrying capacity of prey species, q_1 represents catchability coefficient of prey species, E_1 represents the effort applied to harvest the prey species,the term dy represents the growth rate of predator species due to the predation, c represents death rate of predator species, q_2 represents catchability coefficient of predator species, E_2 represents the effort applied to harvest the predator species. Throughout this analysis we assume that $r-q_1E_1 > 0$ and $d - q_2E_2 > 0$

3. Analysis of steady states

It is well known that the steady states of the system (2.1)-(2.2) are independent of time. The possible steady states are $G_0(0,0)$,$G_1(\bar{x},0)$,$G_2(0,\bar{y})$, $G_3(x^*,y^*)$. In the non appearance of predator species, the possible steady state $G_1(\bar{x},0)$ where

$$\bar{x} = \frac{k(r - q_1E_1)}{r} \tag{3.1}$$

In the non appearance of prey species, the suitable equilibrium point is $G_2(0,\bar{x})$, where

$$\bar{y} = \frac{d - q_2E_2}{c} \tag{3.2}$$

Here both $\bar{x}$ and $\bar{y}$ are positive since $r-q_1E_1 > 0$ and $d-q_2E_2 > 0$. In the presence of both prey and predator species, the steady state point (interior) is $G_3(x^*,y^*)$ where

$$y^* = \frac{(b+x^*)(r - \frac{rx^*}{k} - q_1E_1)}{a} \tag{3.3}$$

For y^* to be positive, we must have $r - q_1E_1 > \frac{rx^*}{k}$. For x^* to be positive, it is required to form a cubic equation from (2.1)-(2.2) by equating the corresponding sides to zero.

$$a_1{x^*}^3 + b_1{x^*}^2 + c_1x^* + d_1 = 0 \tag{3.4}$$

where $a_1 = \frac{cr}{ka} > 0$, $b_1 = \frac{2cbr}{ka} - \frac{d}{k} - \frac{cr}{a} + \frac{cq_1E_1}{a}$, $c_1 = a + d - \frac{2cbr}{a} + 2cbq_1E_1 - q_2E_2 + \frac{cb^2r}{ka} - \frac{db}{k}$, $d_1 = bd - \frac{cb^2r}{a} + \frac{cb^2q_1E_1}{a} - bq_2E_2$.
Equation(3.4) has a unique positive solution if the following inequalities hold
(i) $\frac{2cbr}{ka} + \frac{cq_1E_1}{a} < \frac{d}{k} + \frac{cr}{a}$, (ii) $a + d + 2cbq_1E_1 + \frac{cb^2r}{ka} < \frac{2cbr}{a} + q_2E_2 + \frac{db}{k}$,
(iii) $d + \frac{cbq_1E_1}{a} < \frac{cbr}{a} + q_2E_2$.

4. Local stability analysis

It is nothing but the steadiness of the given ecological system. Since the equilibrium points $G_0(0,0)$,$G_1(\bar{x},0)$,$G_2(0,\bar{y})$ are unstable, the present study is restricted to the inner steady state. Now, we analyze the steadiness, in specific local one. To checked the local steadiness, it is mandatory to custom the typical equation of variational matrix for the system (2.1)-(2.2) around $G_3(x^*,y^*)$ is

$$\lambda^2 + A_1\lambda + A_2 = 0 \tag{4.1}$$

where $A_1 = cy^* + \frac{rx^*}{k} - \frac{ax^*y^*}{(b+x^*)^2} - \frac{dx^*}{k}$; $A_2 = \frac{rcx^*y^*}{k} - \frac{acx^*(y^*)^2}{(b+x^*)^2} - \frac{a^2bx^*y^*}{(b+x^*)^3} + \frac{adx^*y^*}{k(b+x^*)} - \frac{rdx^*y^*}{k^2} + \frac{ad(x^*)^2y^*}{k(b+x^*)^2}$.
Now the stability of the system (2.1)-(2.2) purely depends on the signs of eigenvalues (roots λ_1,λ_2 of the equation (4.1)),and therefore $G_3(x^*,y^*)$ is locally asymptotically stable if $\lambda_1+\lambda_2<0$ and $\lambda_1\lambda_2>0$ provided $r>d$, $c>\frac{ax^*}{(b+x^*)^2}$.

5. Study of Global stability

Theorem(I). The Steady state $G_3(x^*,y^*)$ is globally asymptotically stable.
Proof: Let us consider the Lyapunov function $v(x,y) = \left[x - x^* - x^*\ln\left(\frac{x}{x^*}\right)\right] + l_1\left[y - y^* - y^*\ln\left(\frac{y}{y^*}\right)\right]$.

$$\begin{aligned}
\frac{dv}{dt} &= \left(\frac{x-x^*}{x}\right)\frac{dx}{dt} + l_1\left(\frac{y-y^*}{y}\right)\frac{dy}{dt} \\
&= (x-x^*)\left[r - \frac{rx}{k} - \frac{ay}{b+x} - q_1E_1\right] + l_1(y-y^*)\left[\frac{ax}{b+x} + d\left(1-\frac{x}{k}\right) - cy - q_2E_2\right] \\
&= (x-x^*)\left[\frac{-r}{k}(x-x^*) - a\left(\frac{y}{b+x} - \frac{y^*}{b+x^*}\right)\right] + l_1(y-y^*)\left[a\left(\frac{x}{b+x} - \frac{x^*}{b+x^*}\right)\right. \\
&- \left.\frac{d}{k}(x-x^*) - c(y-y^*)\right].
\end{aligned}$$

Choose, $l_1 = \frac{b+x^*}{b}$

$$\begin{aligned}
\frac{dv}{dt} &= \frac{-r}{k}(x-x^*)^2 - \frac{ay^*(x-x^*)^2}{(b+x)(b+x^*)} - \frac{d(b+x^*)(x-x^*)(y-y^*)}{kb} - \frac{c(b+x^*)(y-y^*)^2}{b} \\
&\le -\left[\frac{r}{k} + \frac{ay^*}{(b+x)(b+x^*)}\right](x-x^*)^2 + \left[\frac{d(b+x^*)}{2kb}\right](x-x^*)^2 \\
&+ \left[\frac{d(b+x^*)}{2kb}\right](y-y^*)^2 - \left[\frac{c(b+x^*)}{b}\right](y-y^*)^2
\end{aligned}$$

So, $\frac{dv}{dt} < -\left[\frac{r}{k} + \frac{ay^*}{(b+x)(b+x^*)} + \frac{d(b+x^*)}{2kb}\right](x-x^*)^2 + \left[\frac{d(b+x^*)}{2kb} - \frac{c(b+x^*)}{b}\right](y-y^*)^2$.
Thus we conclude that $\frac{dv}{dt}<0$ provided the coefficients of $(x-x^*)^2$ and $(y-y^*)^2$ are negative. Hence the system is globally asymptotically stable if the conditions $(i)\left[\frac{r}{k} + \frac{ay^*}{(b+x)(b+x^*)}\right] > \frac{d(b+x^*)}{2kb}$ and $(ii)\frac{d}{2c} < k$ hold.

6. Analysis of bionomic equilibria

It is the study of the dynamics of living resources using economic models. Let c_1 be the fishing cost per unit effort for prey species, c_2 be the fishing cost per unit effort for predator species, p_1 be the price per unit biomass of the prey, p_2 be the price per unit biomass of the predator. Therefore net revenue or economic rent at any time given by $R = R_1 + R_2$ where $R_1 = (p_1 q_1 x - c_1)\, E_1,\ R_2 = (p_2 q_2 y - c_2)\, E_2$, where R_1 represents Net Revenue for the prey and R_2 represents Net revenue for predator species. The bionomic equilibrium $(x_\infty, y_\infty, (E_1)_\infty, (E_2)_\infty)$ is given by the following equations

$$rx\left(1 - \frac{x}{k}\right) - \frac{axy}{b+x} - q_1 E_1 x = 0, \tag{6.1}$$

$$\frac{axy}{b+x} + dy\left(1 - \frac{x}{k}\right) - cy^2 - q_2 E_2 y = 0 \tag{6.2}$$

$$R = (p_1 q_1 x - c_1)\, E_1 + p_2 q_2 y - c_2) E_2 = 0 \tag{6.3}$$

In order to determine the bionomic equilibrium we now consider the following cases.
Case(I): If $c_2 > p_2 q_2 y$ i.e., the cost is greater than the revenue for the predator, then the predator fishing will be stopped($E_2 = 0$).Only the prey fishing remains operational($c_1 < p_1 q_1 x$) we then have $x_\infty = \frac{c_1}{p_1 q_1}$.
Case(II): If $c_1 > p_1 q_1 x$ i.e., the cost is greater than the revenue in the prey fishing,then the prey fishing will be closed ($E_1 = 0$).Only the predator fishing remains operational($c_2 < p_2 q_2 y$), then we have $y_\infty = \frac{c_2}{p_2 q_2}$.
Case(III): If $c_1 > p_1 q_1 x$ and $c_2 > p_2 q_2 y$ i.e., the cost is greater than the revenues for the both the species then the whole fishery will be closed.
Case(IV): If $c_1 < p_1 q_1 x$ and $c_2 < p_2 q_2 y$ i.e., the revenues for the both the species being positive.Then the whole fishery is in operation. In this case $x_\infty = \frac{c_1}{p_1 q_1}$ and $y_\infty = \frac{c_2}{p_2 q_2}$.
Using x_∞, y_∞ and from (6.1),(6.2),we get

$$(E_1)_\infty = \frac{1}{q_1}\left[r - \frac{rc_1}{kp_1 q_1} - \frac{ac_2}{p_2 q_2\left(b + \frac{c_1}{p_1 q_1}\right)}\right] \tag{6.4}$$

$$(E_2)_\infty = \frac{1}{q_2}\left[\frac{ac_1}{p_1 q_1 b + c_1} + d - \frac{dc_1}{p_1 q_1 k} - \frac{cc_2}{p_2 q_2}\right]. \tag{6.5}$$

For $(E_1)_\infty$ and $(E_2)_\infty$ are to be positive, provided

$$r\left(1 - \frac{M}{k}\right) > \frac{aN}{b+M} \tag{6.6}$$

and

$$d\left(1 - \frac{M}{k}\right) > cN - \frac{aM}{b+M} \tag{6.7}$$

where $M = \frac{c_1}{p_1 q_1},\ N = \frac{c_2}{p_2 q_2}$. Thus the Non-trivial bionomic equilibrium point $(x_\infty, y_\infty, (E_1)_\infty, (E_2)_\infty)$ exists if the conditions (6.6) and (6.7) hold.

7. Optimal harvesting strategy

Here the objective is to maximize the present value J of a continuous time stream of revenues given by

$$J = \int_0^\infty e^{-\delta t} \left[\left(p_1 q_1 x - c_1 \right) E_1 + (p_2 q_2 y - c_2) E_2 \right] dt \tag{7.1}$$

where δ denotes the instantaneous annual rate of discount. Now our problem is to maximize J subject to the state equations (2.1) and (2.2) by invoking Pontryagin's maximum principle.The control variable $E_i (i = 1, 2)$ are subjected to the constraints

$$0 \leq E_i \leq (E_i)_{max} \tag{7.2}$$

The Hamiltonian for the problem is given by

$$H = e^{-\delta t} \left[\left(p_1 q_1 x - c_1 \right) E_1 + \left(p_2 q_2 y - c_2 \right) E_2 \right] + \lambda_1 \left[rx \left(1 - \frac{x}{k} \right) - \frac{axy}{b + x} - q_1 E_1 x \right]$$
$$+ \lambda_2 \left[\frac{axy}{b + x} + dy \left(1 - \frac{x}{k} \right) - cy^2 - q_2 E_2 y \right] \tag{7.3}$$

where λ_1 and λ_2 are the adjoint variables.The control variables $E_1$and E_2 appear linearly in the Hamiltonian function H. Let the control constraints are not binding i.e. the optimal solution does not occur at $(E_i)_{max}$, we have singular control. According to Pontryagin's maximum principle,

$$\frac{\partial H}{\partial E_1} = 0, \frac{\partial H}{\partial E_2} = 0, \frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x}, \frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y}. \tag{7.4}$$

Now,from $\frac{\partial H}{\partial E_1} = 0$ and $\frac{\partial H}{\partial E_2} = 0$ the values of λ_1, λ_2 are

$$\lambda_1 = e^{-\delta t} \left[p_1 - \frac{c_1}{q_1 x} \right] \tag{7.5}$$

$$\lambda_2 = e^{-\delta t} \left[p_2 - \frac{c_2}{q_2 y} \right] \tag{7.6}$$

Again, from $\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x}$ and $\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y}$, we have

$$\frac{d\lambda_1}{dt} = - \left[e^{-\delta t} p_1 q_1 E_1 + \lambda_1 \left[r - \frac{2rx}{k} - \frac{aby}{(b+x)^2} - q_1 E_1 \right] + \lambda_2 \left[\frac{aby}{(b+x)^2} - \frac{dy}{k} \right] \right] \tag{7.7}$$

$$\frac{d\lambda_2}{dt} = - \left[e^{-\delta t} p_2 q_2 E_2 + \lambda_1 \left(-\frac{ax}{b+x} \right) + \lambda_2 \left(\frac{ax}{b+x} + d - 2cy - q_2 E_2 \right) \right] \tag{7.8}$$

From (7.5) and (7.8), we get a linear equation which is in the form of $\frac{d\lambda_2}{dt} + \lambda_2 A_1 = e^{-\delta t} A_2$, and whose solution is given by

$$\lambda_2 = e^{-\delta t} \left[\frac{A_2}{A_1 - \delta} \right], \tag{7.9}$$

where $A_1 = \frac{ax}{b+x} + d - 2cy - q_2E_2,\ A_2 = \left(x - \frac{c_1}{p_1q_1}\right)\frac{ax}{b+x} - p_2q_2E_2$. From (7.6) and (7.7) we get a linear differential equation which is in the form of $\frac{d\lambda_1}{dt} + \lambda_1 B_1 = e^{-\delta t}B_2$ and whose solution is given by

$$\lambda_1 = e^{-\delta t}\left[\frac{B_2}{B_1 - \delta}\right], \tag{7.10}$$

where

$$B_1 = r - \frac{2rx}{k} - \frac{aby}{(b+x)^2} - q_1E_1,\ B_2 = \left[-p_1q_1E_1 - \left(p_2 - \frac{c_2}{q_2y}\right)\left(\frac{aby}{(b+x)^2} - \frac{dy}{k}\right)\right].$$

From (7.9) and (7.10) we observe that the shadow price $\lambda_i(t)e^{\delta t}, i = 1,2$ remain constant over time in optimal equilibrium when they remain bounded as $t \to \infty$.

8. Diffusion analysis

In this section, the spatial heterogeneity system with harvesting and diffusion is proposed and investigated. The proposed mathematical model is given by

$$\frac{\partial x}{\partial t} = rx\left(1 - \frac{x}{k}\right) - \frac{axy}{b+x} - q_1E_1x + D_1x_{uu} \tag{8.1}$$

$$\frac{\partial y}{\partial t} = \frac{axy}{b+x} + dy\left(1 - \frac{x}{k}\right) - cy^2 - q_2E_2y + D_2y_{uu} \tag{8.2}$$

In this segment, we deliberated the exceptional influences of transmission of the ideal structure (8.1)-(8.2). Let $x = x(u,t), y = y(u,t)$, where u is a space variable and $x(u,0) > 0$; $y(u,0) > 0$; for $u \in [0, \mathrm{L}]$. The trivial fluctuation edge conditions are specified by $[x_u]_{u=0,L} = 0; [y_u]_{u=0,L} = 0$. Now, let us consider the ideal (8.1)-(8.2) underneath trivial fluctuations edge ailments. To analyze the role of transmission on this model, we deliberate the linear model of the structure (8.1)-(8.2) about the interior steady state $G_3(x^*, y^*)$ as given by

$$X'(t) = -\frac{rx^*X}{k} + D_1X_{uu} \tag{8.3}$$

$$Y'(t) = -\frac{dy^*X}{k} - cy^*Y + D_2Y_{uu} \tag{8.4}$$

by putting $x = x^* + X$; $y = y^* + Y$;. Assume the solutions of equations (8.3)-(8.4) are in the form

$$X = \alpha_1 e^{\lambda t}\cos pu; Y = \alpha_2 e^{\lambda t}\cos pu \tag{8.5}$$

where p is the wave numeral of perturbation, λ is the frequency numeral & $\alpha_i, i = 1,2$ are the amplitudes. The characteristic equation of (8.3)-(8.4) using (8.5) is

$$\mu^2 + A\mu + B = 0 \tag{8.6}$$

where $A = \frac{rx^*}{k} + cy^* + (D_1 + D_2)p^2$; $B = \frac{rcx^*y^*}{k} + p^2(\frac{rx^*D_2}{k} + cy^*D_1) + D_1D_2p^4$
Now, our main aim is to find the ailments for diffusive unsteadiness of model system (8.1)-(8.2), for this, let us rewrite B as $\phi(p^2)$ where
$\phi(p^2) = D_1D_2(p^2)^2 + p^2(\frac{rx^*D_2}{k} + cy^*D_1) + \frac{rcx^*y^*}{k}$.
The system (8.1)-(8.2) is unstable if one of the above roots of the equation (8.6)

Table 1. The hypothetical set of parameter values.

Parameters	values
r	2.5
k	10
a	0.25
b	1.2
q_1	.3
d	2
c	0.15
q_2	0.21
E_1	0.14
E_2	0.16

is optimistic. A necessary and sufficient condition for a root to be positive is that $\frac{rx^*}{k} + cy^* + (D_1 + D_2)p^2 > 0$. The sufficient condition for positivity of one of the roots of the equation (8.6) is $\phi(p^2) < 0$. Since $\phi(p^2)$ is an expression in p^2 where p the wave number, non zero positive quantity, the minimum of $\phi(p^2)$ occurs. Let $(p^2)_{min}$ be the corresponding value of p^2 for minimum value of $\phi(p^2)$, then $(p^2)_{min} = -\frac{\frac{rx^* D_2}{k} + cy^* D_1}{D_1 D_2} > 0$. The corresponding minimum value of $\phi(p^2)$ is $\phi(p^2)_{min} = \frac{\frac{rx^* D_2}{k} + cy^* D_1}{4D_1 D_2} < \frac{rcx^* y^*}{k}$, provided $\Gamma > \frac{rx^*}{kcy^*}$ where $\Gamma = \frac{D_1}{D_2}$

9. Numerical simulations

With the parameter values given in the table 1, we established the analytical findings through numerical simulations using MATLAB. It is also observed that Increase in predation rate ($a = 0.75$) cause extinction of the prey population.Further change in the growth rate of prey species the system shows shift in the equilibrium point where one of the coexistence steady state collide with an axial equilibrium point.

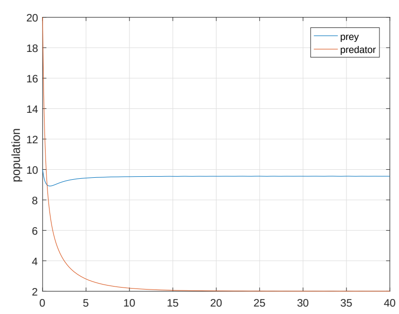

Figure 1. represents the variation of species against time't'

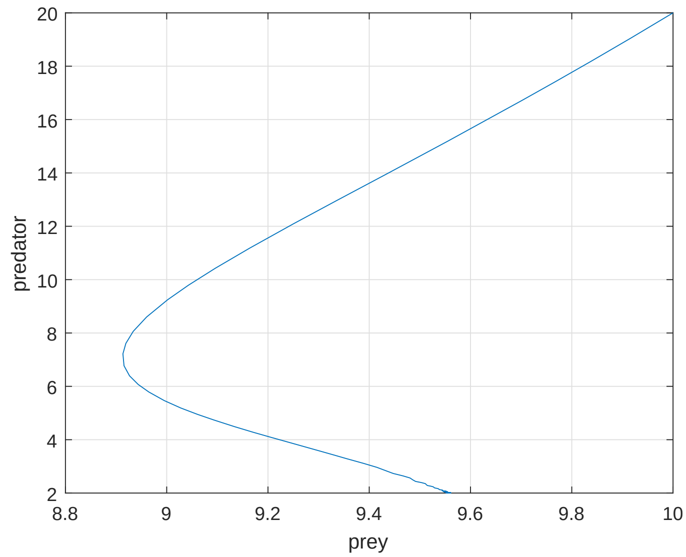

FIGURE 2. represents the phase portrait diagram of species prey and predator.

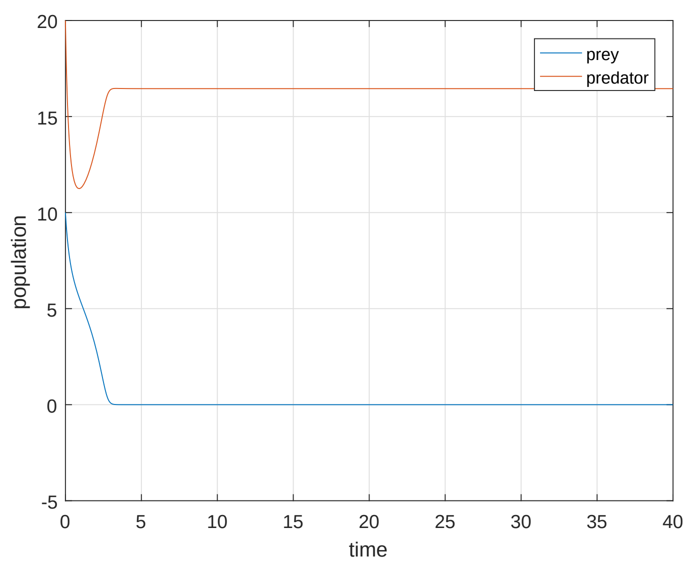

FIGURE 3. represents the variation of species against time't' with $a = 0.75$

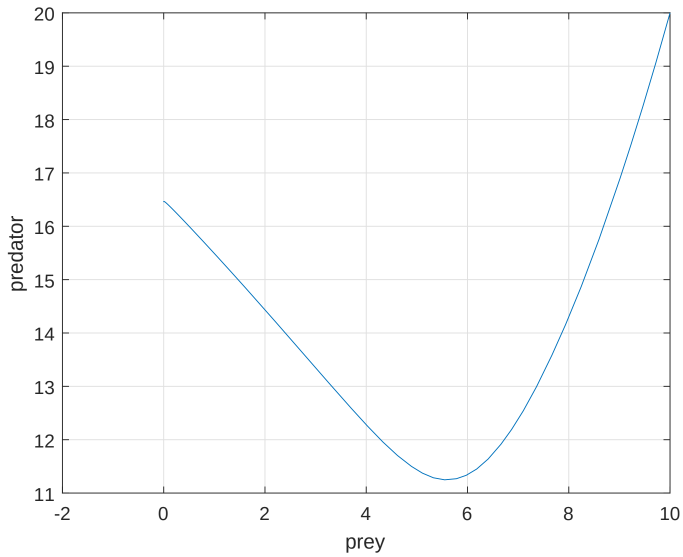

FIGURE 4. represents the phase portrait diagram of species prey and predator with $a = 0.75$.

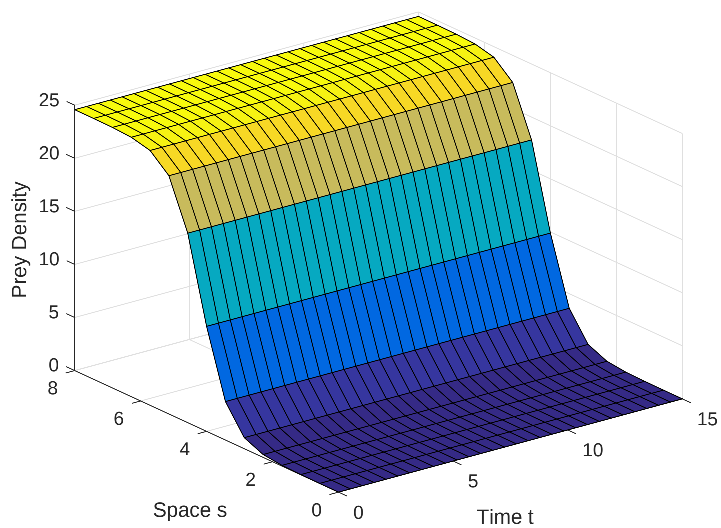

FIGURE 5. denotes the steady fluctuations of the prey species against space and time with $D_1 = 0.1, D_2 = 0.2$.

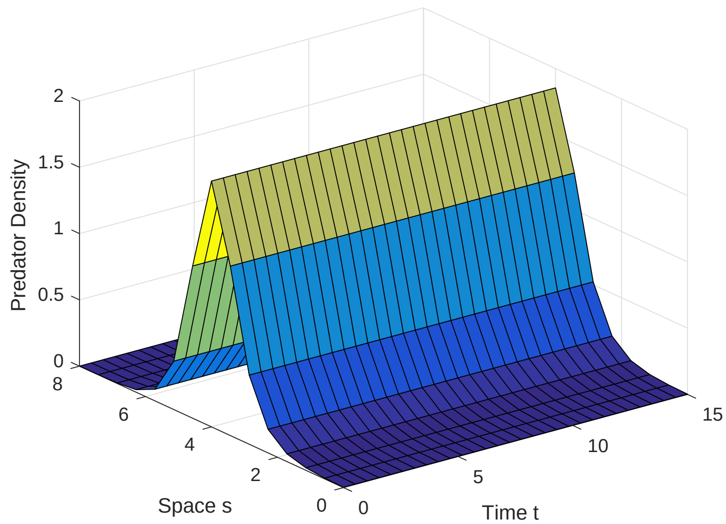

FIGURE 6. denotes the steady fluctuations of the predator species against space and time with $D_1 = 0.1, D_2 = 0.2$

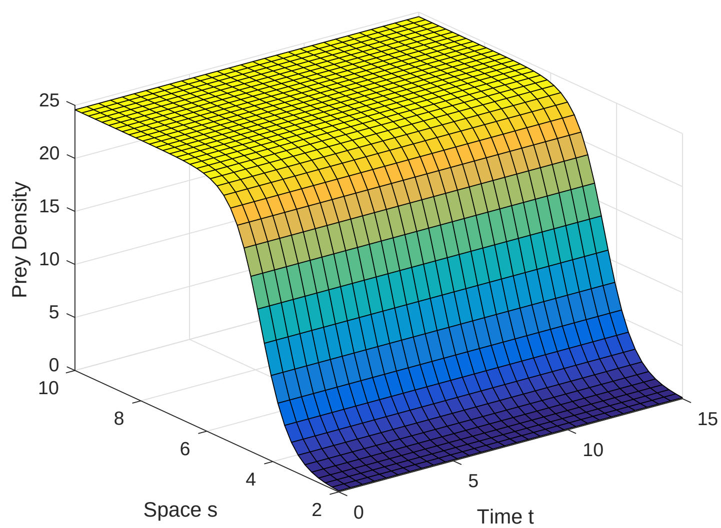

FIGURE 7. denotes the steady fluctuations of the prey species against space and time with $D_1 = 0.001, D_2 = 0.002$.

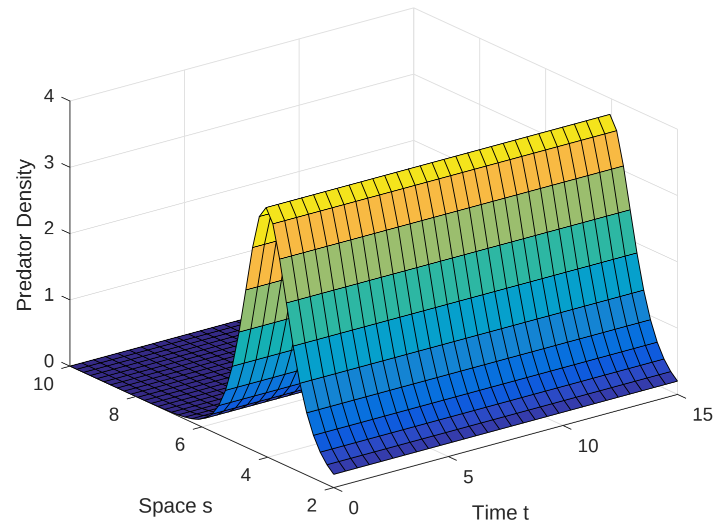

FIGURE 8. denotes the steady fluctuations of the predator species against space and time with $D_1 = 0.001, D_2 = 0.002$

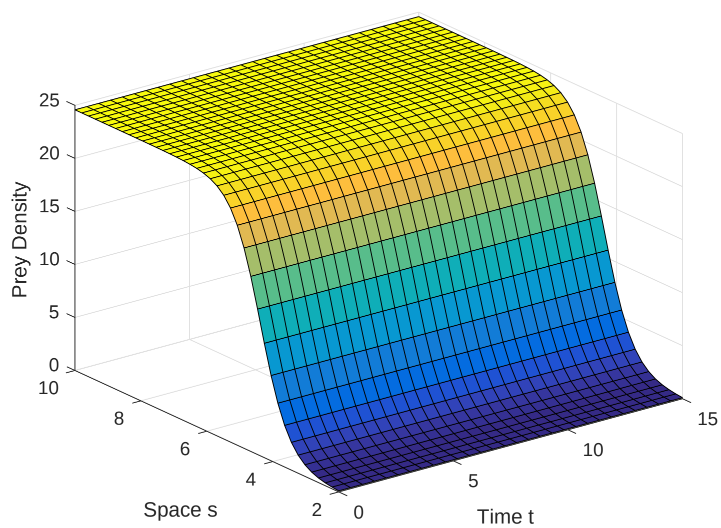

FIGURE 9. denotes the steady fluctuations of the prey species against space and time with $D_1 = 10, D_2 = 20$.

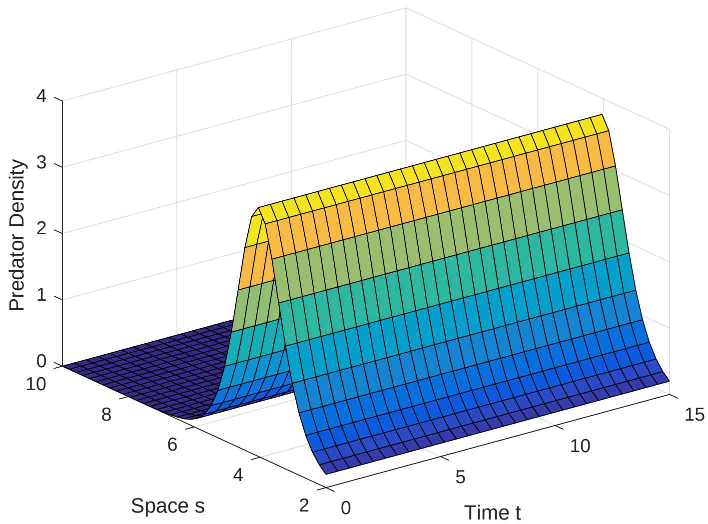

FIGURE 10. denotes the steady fluctuations of the predator species against space and time with $D_1 = 10, D_2 = 20$

10. Concluding remarks

This research article mainly concentrates and aims at the most interesting changing aspects of a prey, predator trophic interaction model with harvesting with Holling type-II functional response. We obtain the possible equilibrium points and analyzed. Stability in terms of local and global is checked in view of R-H Criteria and suitable construction of Lyapunov function. Bio-economic and feasible harvesting strategies have been computed using maximum principle. It is shown that the dynamics of deterministic system in the figures (1)-(2). we also verified the steadiness of the system (2.1)-(2.2) in the figures (3)-(4) by increasing the predation rate. Also analysed instability condition for diffusive structure of the ideal structure (8.1)-(8.2). It is also verified the stable oscillations of the prey and predator populations against time and space in figures (5)-(10).

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