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FINITE DIFFERENCE SCHEME FOR SPACE FRACTIONAL DIFFUSION EQUATION WITH MIXED BOUNDARY CONDITIONS

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***Abstract*—The aim of this paper is to develop an explicit finite difference scheme for space fractional diffusion equation (SFDE) with mixed initial boundary conditions. We also show that the scheme is conditionally stable and convergent. As an application of the scheme, numerical solution for space fractional diffusion equation is obtained with the help of Mathematica software.**

Keywords: Fractional diffusion equation, Fractional derivative, Difference scheme, Mathematica.

I. INTRODUCTION

The philosopher and creator of modern calculus G.W. Leibniz made some remarks on the meaning and possibility of fractional derivative of order $\frac{1}{2}$ in the late seventeenth century. A rigorous investigation was first carried out by Liouville in a series of papers from 1832-1837, where he defined the fist outcast of an operator of fractional integration. A number of authors have recently begun to study space fractional diffusion equations. The basic analytical theory for the space fractional diffusion processes had been developed in 1952 by Fell. Recently, fractional derivatives have found new applications in engineering, physics, finance and hydrology [1, 2, 3]. The theory of fractional calculus is a useful mathematical tool for applied sciences but it is hard to tackle. Now days some different numerical methods for solving the fractional diffusion equations are available in the literature [4, 5]. Numerical methods and theoretical analysis of fractional differential equations are still at an early stage of development. Therefore, we developed the space fractional order explicit finite difference scheme (SFOEFDS) for diffusion equation with mixed initial boundary conditions. Finally, some numerical examples are to prove that the numerical results are in good agreement with our theoretical analysis.

We consider the space fractional diffusion equation (SFDE) with mixed initial boundary conditions:

$$\frac{\partial u(x,t)}{\partial t} = d(x)\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} + q(x,t),\ 0 < x < L,\ 1 < \alpha < 2 \tag{1}$$

$$initial\ condition : u(x,0) = \phi(x), 0 \le x \le L \tag{2}$$

$$boundary\ conditions : -\frac{\partial u(0,t)}{\partial x} + u(0,t) = 0,$$

$$\frac{\partial u(L,t)}{\partial x} + u(L,t) = 0, t > 0 \tag{3}$$

where variable coefficient $d(x) > 0$. Here, $\frac{\partial^\alpha u(x,t)}{\partial x^\alpha}$ is Caputo's fractional derivative ${}_oD_x^\alpha u(x)$ which is defined as

$$\frac{\partial^\alpha u}{\partial x^\alpha} ={}_o D_x^\alpha u(x) = \frac{1}{\Gamma(m-\alpha)}\int_0^x (x-\xi)^{m-\alpha-1}\,\frac{d^m u(\xi)}{d\xi^m}\,d\xi,$$

$$m-1 < \alpha < m$$

$$\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} = \frac{d^m u(x)}{dx^m},\ \ \alpha = m\ \varepsilon\ N.$$

where $\Gamma(.)$ is the gamma function.

We organize this paper as follows: In section 2, the explicit finite difference scheme is developed for nonhomogeneous space fractional diffusion equation. The section 3, is devoted for stability of scheme and the question of convergence is proved in section 4. Numerical solution of space fractional diffusion equation is obtained by using Mathematica software in the last section.

II. EXPLICIT FINITE DIFFERENCE SCHEME FOR SFDE

In this section we develop the space fractional explicit finite difference scheme (SFEFDS) for fractional diffusion equation (FDE) $(1)-(3)$. Consider $h = \frac{x}{k}$, k is positive integer, we use the second order difference approximation, we get

$$\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} ={}_o D_x^\alpha u(x,t) = \frac{1}{\Gamma(2-\alpha)}\int_0^x \frac{1}{(x-\xi)^{\alpha-1}}\,\frac{\partial^2 u(\xi,t)}{\partial\xi^2}\,d\xi,$$

$$= \frac{1}{\Gamma(2-\alpha)} \sum_{j=0}^{N-1} \int_{jh}^{(j+1)h} \eta^{1-\alpha} \frac{\partial^2 u(x-\eta,t)}{\partial \eta^2} d\eta$$

$$\approx \tfrac{1}{\Gamma(2-\alpha)} \sum_{j=0}^{k-1} [\tfrac{u(x-(j-1)h,t)-2u(x-jh,t)+u(x-(j+1)h,t)}{h^2}]$$

$$\times \int_{jh}^{(j+1)h} \eta^{1-\alpha} d\eta$$

$$= \frac{h^{-\alpha}}{\Gamma(3-\alpha)} \sum_{j=0}^{k-1} [u(x-(j-1)h,t) - 2u(x-jh,t)$$
$$+u(x-(j+1)h,t)][(j+1)^{2-\alpha} - j^{2-\alpha}]$$

Let $\Delta t = \tau$ be the grid step size in time, $t_n = n\Delta t = n\tau$, $0 \leq t_n \leq T$, $\tau = \frac{T}{n}$ and $\Delta x = h$ be the grid step size in space, $x_j = jh$, $0 \leq x_j \leq L$, for j = 0, 1, 2, ... , N, $Nh = L$. Let $u_0^n = u(0, n\tau)$,

$$u_1^n = u(h, n\tau), ..., u_{k-j}^n = u((k-j)h, n\tau), ..., u_j^n = u(jh, n\tau),$$

$$d_j = d(x_j), \phi_j = \phi(x_j).$$

We approximate SFDE (1), by using an explicit finite difference approximation (EFDA), we get

$\frac{u_k^{n+1} - u_k^n}{\tau}$

$= \frac{d_k}{\Gamma(3-\alpha)} \sum_{j=0}^{k-1} \frac{[u_{k-j+1}^n - 2u_{k-j}^n + u_{k-j-1}^n]}{h^\alpha} [(j+1)^{2-\alpha} - j^{2-\alpha}] + q_k^n$

$\frac{u_k^{n+1} - u_k^n}{\tau}$

$= \frac{d_k h^{-\alpha}}{\Gamma(3-\alpha)} \sum_{j=0}^{k-1} [u_{k-j+1}^n - 2u_{k-j}^n + u_{k-j-1}^n][(j+1)^{2-\alpha} - j^{2-\alpha}]$

$+ q_k^n$

After simplification , we get

$$u_k^{n+1} = b_k u_{k+1}^n + (1-2b_k)u_k^n + b_k u_{k-1}^n$$
$$+b_k \sum_{j=1}^{k-1} g_j [u_{k-j+1}^n - 2u_{k-j}^n + u_{k-j-1}^n] + \tau q_k^n$$

where $b_k = \frac{\tau d_k}{h^\alpha \Gamma(3-\alpha)}$, $and\ g_k = (j+1)^{2-\alpha} - j^{2-\alpha}$.
The initial condition is approximated as $u_j^0 = \phi(x_j)$,
j = 0, 1, 2, ... , N.
Using central difference the boundary conditions are approximated as

$$-\frac{u_1^n - u_{-1}^n}{2h} + u_0^n = 0,\ and\ \frac{u_{N+1}^n - u_{N-1}^n}{2h} + u_N^n = 0,$$
$$implies\ u_{-1}^n = u_1^n - 2hu_0^n\ and\ u_{N+1}^n = u_{N-1}^n - 2hu_N^n$$

Therefore, the fractional approximated IBVP is

$$u_k^{n+1} = b_k u_{k+1}^n + (1-2b_k)u_k^n + b_k u_{k-1}^n$$
$$+b_k \sum_{j=1}^{k-1} g_j [u_{k-j+1}^n - 2u_{k-j}^n + u_{k-j-1}^n] + \tau q_k^n \qquad (4)$$

$$initial\ condition:\ u_j^0 = \phi(x_j), j = 0, 1, 2, ..., N. \qquad (5)$$

$$boundary\ conditions:\ -\frac{u_1^n - u_{-1}^n}{2h} + u_0^n = 0,$$
$$\frac{u_{N+1}^n - u_{N-1}^n}{2h} + u_N^n = 0 \qquad (6)$$

where

$$b_k = \frac{\tau d_k}{h^\alpha \Gamma(3-\alpha)},\ and\ g_k = (j+1)^{2-\alpha} - j^{2-\alpha}.$$

Therefore, the above IBVP (4) – (6) can be written in the following matrix equation:

$$U^{n+1} = AU^n + B \qquad (7)$$

where

$$U^n = (u_0^n, u_1^n, ..., u_N^n)^T$$

and $A = (a_{ij})$ is a square matrix of coefficient of order N. For $i = 0, 1, 2, ..., N,\ j = 0, 1, 2, ..., N$ the coefficients are

$$a_{ij} = \begin{cases} 0,\ when\ j \geq i+2 \\ b_i,\ when\ j = i+1 \\ 1 - b_i(2-g_1),\ \ when\ j = i = 3, 4, ..., N \\ b_i(1 - 2g_1 + g_2),\ when\ j = i-1,\ i = 3, 4, ..., N-1 \\ b_i(g_{i-j-1} - 2g_{i-j} + g_{i-j+1}),\ when\ j \leq\ i-2 \end{cases} \qquad (8)$$

while $a_{00} = 1 - 2b_0(1+h), a_{01} = 2b_0, a_{10} = b_1, a_{11} = 1 - 2b_1, a_{i0} = b_i g_{i-1},\ 2 \leq i \leq N,\ a_{i1} = b_i(g_{i-2} - 2g_{i-1}),\ 3 \leq i \leq N, a_{N9} = b_{10}(2 - 2g_1 + g_2),\ a_{NN} = 1 - b_{10}(2 + 2h - g_1)$ and

$$B = [\tau q_0^n, \tau q_1^n, \tau q_2^n, ..., \tau q_{N-1}^n, \tau q_N^n]^T$$

The above system of algebraic equations is solved by using Mathematica software in section 5.

III. STABILITY

This section is devoted for the stability criteria of the space fractional explicit finite difference scheme (4) – (6) for the SFDE (1) – (3).

Theorem 3.1: The solution of the space fractional explicit finite difference scheme (SFEFDS) (4)−(6) for SFDE (1)−(3) is conditionally stable.
Proof: Consider the equation (7), we have

$$U^{n+1} = AU^n + B, \qquad (9)$$

By Gerschgorin's first theorem [6], let λ_i be an eigenvalue of the matrix A to linear system of equations (7), and x be the corresponding eigenvector then $Ax = \lambda x$.
Choose i such that $|x_i| = max\{|x_j|, j = 0, 1, ..., N\}$, then $\sum_{j=0}^{N} a_{ij} x_j = \lambda x_i$, and therefore,

$$\lambda = a_{ii} + \sum_{j=0, j \neq i}^{N} a_{ij} \frac{x_j}{x_i} \qquad (10)$$

We substitute the values of a_{ij} in equation (10), we get
(i) When i = 0,

$$\lambda = 1-2b_0(1+h)+2b_0\frac{x_1}{x_0}$$

$$\lambda \le 1-2b_0-2b_0h+2b_0 = 1-2b_0h \le 1$$

$$\lambda = 1-2b_0(1+h)+2b_0\frac{x_1}{x_0} \ge 1-2b_0(2+h)$$

If $b_0(2+h) \le 1$ *implies* $b_0 \le \frac{1}{(2+h)}$ then

$$-1 \le \lambda \le 1 \ i.e.\ |\lambda| \le 1.$$

(ii) When i = 1,

$$\lambda = 1-2b_1+b_1\frac{x_0}{x_1}+b_1\frac{x_2}{x_1} \le 1$$

$$\lambda = 1-2b_1+b_1\frac{x_0}{x_1}+b_1\frac{x_2}{x_1} \ge 1-4b_1$$

If $2b_1 \le 1$ *implies* $b_1 \le \frac{1}{2}$ then

$$-1 \le \lambda \le 1 \ i.e.\ |\lambda| \le 1.$$

(iii) When i = 2,

$$\lambda = 1-2b_2(2-g_1)+b_2g_1\frac{x_0}{x_2}+b_2(1-2g_1)\frac{x_1}{x_2}+b_2\frac{x_3}{x_2} \le 1$$

$$\lambda = 1-2b_2(2-g_1)-b_2g_1\frac{x_0}{x_2}-b_2(1-2g_1)\frac{x_1}{x_2}-b_2\frac{x_3}{x_2}$$
$$\ge 1-2b_2(2-g_1)$$

If $b_2(2-g_1) \le 1$ *implies* $b_2 \le \frac{1}{(2-g_1)}$ then

$$-1 \le \lambda \le 1 \ i.e.\ |\lambda| \le 1.$$

(iv) When i = 3,

$$\lambda = 1-b_3(2-g_1)+b_3g_2\frac{x_0}{x_3}+b_3(g_1-2g_2)\frac{x_1}{x_3}$$
$$+b_3(1-2g_1+g_2)\frac{x_2}{x_3}+b_3\frac{x_4}{x_3} \le 1$$

$$\lambda = 1-b_3(2-g_1)-b_3g_2\frac{x_0}{x_3}-b_3(g_1-2g_2)\frac{x_1}{x_3}$$
$$-b_3(1-2g_1+g_2)\frac{x_2}{x_3}-b_3\frac{x_4}{x_3} \ge 1-2b_3(2-g_1-g_2)$$

If $b_3(2-g_1-g_2) \le 1$ *implies* $b_3 \le \frac{1}{(2-g_1-g_2)}$ then

$$-1 \le \lambda \le 1 \ i.e.\ |\lambda| \le 1.$$

(v) When $4 \le i \le N-1$,

$$\lambda = 1-b_i(2-g_1)+b_ig_{i-1}\frac{x_{i-4}}{x_i}+b_i(g_{i-2}-2g_{i-1})\frac{x_{i-3}}{x_i}$$
$$+b_i\sum_{j=2}^{i-1}(g_{i-j-1}-2g_{i-j}+g_{i-j+1})\frac{x_{i-2}}{x_i}$$
$$+b_i(1-2g_1+g_2)\frac{x_{i-1}}{x_i}+b_i\frac{x_{i+1}}{x_i} \qquad (11)$$

We note that $g_i > g_{i+1},\ g_{i-j-1}-2g_{i-j}+g_{i-j+1} > 0$ for $j = 0,1,2,...,i-1,$ and $i = 0,1,2,...,N-1$

$$\sum_{j=2}^{i-1}(g_{i-j-1}-2g_{i-j}+g_{i-j+1}) \le g_{i-1}-g_{i-2}+g_0-g_1.$$

Since b_i are non-negative real numbers, then from equation (11), we get

$$\lambda \le 1-b_i(2-g_1)+b_ig_{i-1}+b_i(g_{i-2}-2g_{i-1})+b_i(g_{i-1}-g_{i-2}+g_0-g_1)$$
$$+b_i(1-2g_1+g_2)+b_i \le 1$$

$$\lambda \ge 1-b_i(2-g_1)-b_ig_{i-1}-b_i(g_{i-2}-2g_{i-1})-b_i(g_{i-1}-g_{i-2}+g_0-g_1)$$
$$-b_i(1-2g_1+g_2)-b_i$$
$$\ge 1-b_i(4-2g_1+g_2)$$

If $b_i(4+4g_1-g_2-g_0) \le 1$ implies

$$b_i \le \frac{2}{(4+4g_1-g_2-g_0)} \ then \ -1 \le \lambda \le 1 \ i.e.\ |\lambda| \le 1.$$

(vi) When $i = N$

$$\lambda = 1-b_N(2+2h-g_1)+b_Ng_{N-1}\frac{x_0}{x_N}+b_N(g_{N-2}-2g_{N-1})\frac{x_1}{x_N}$$
$$+b_N\sum_{j=2}^{i-1}(g_{i-j-1}-2g_{i-j}+g_{i-j+1})\frac{x_{j-2}}{x_N}+b_N(2-2g_1+g_2)\frac{x_{N-1}}{x_N}$$

$$\lambda \le 1-b_N(2+2h-g_1)+b_Ng_{N-1}+b_N(g_{N-2}-2g_{N-1})$$
$$+b_N(g_{i-1}-g_{i-2}+g_0-g_1)+b_N(2-2g_1+g_2) \le 1$$

$$\lambda \ge 1-b_N(2+2h-g_1)-b_Ng_{N-1}-b_N(g_{N-2}-2g_{N-1})$$
$$-b_N(g_{i-1}-g_{i-2}+g_0-g_1)-b_N(2-2g_1+g_2)$$
$$\ge 1-b_N(4+2h-4g_1+g_0+g_2)$$

If $b_N(4+2h-4g_1+g_2-g_0) \le 1$ implies

$$b_N \le \frac{1}{(4+2h-4g_1+g_2-g_0)} \text{then } -1 \le \lambda \le 1 \ i.e.\ |\lambda| \le 1.$$

Therefore from (i)-(vi), we have prove that if

$$\max_{0 \le i \le N-1}\{b_0,b_1,b_2,b_3,b_i,b_N\} \le 1$$

then the spectral radius $\rho(A)$ of the matrix satisfies $\rho(A) \le 1$.

If

$$\max_{0 \le i \le N-1}\{b_0,b_1,b_2,b_3,b_i,b_N\} \le 1$$

then there exists a positive number $\varepsilon \le C\tau$ such that $\|A\|_m \le \rho(A)+C\tau \le 1+O(\tau)$.
Therefore this prove that the finite difference scheme is conditionally stable.
Hence proof is completed.

IV. CONVERGENCE

In this section we discuss the question of convergence. Consider the another vector

$$\bar{U}^n = [u(x_0,t_n),...,u(x_j,t_n),...,u(x_N,t_n)]^T$$

whose size is N+1, which represents the exact solution at time level t_n. The finite difference scheme (9) becomes

$$\bar{U}^{n+1} = A\bar{U}^n + B + \tau^n \tag{12}$$

where τ^n is the vector of the truncation errors at level t_n.

Theorem 4.1: If $\Lambda \le 1$, where

$$\Lambda = \max_{0\le i\le N-1}\{b_0,b_1,b_2,b_3,b_i,b_N\}$$

then the fractional explicit finite difference scheme (4) – (6) for SFDE (1) – (3) is convergent.
Proof: Let Ω be the region $0<x<L$, $0<t<T$.
Take $(x_j,t_n) = (j\Delta x, n\Delta t)$ for $j = 0,1,...,N$ and $n = 0,1,...,M$ with $N\Delta x = L$, $M\Delta t = T$. We introduce the vector

$$\bar{U}^n = [u(x_0,t_n),...,u(x_j,t_n),...,u(x_N,t_n)]^T$$

satisfying the finite difference scheme (9), We get

$$\bar{U}^{n+1} = A\bar{U}^n + B + \tau^n \tag{13}$$

where τ^n is the vector of the truncation errors at level t_n,

$$\bar{u}_j^0 = \phi(x_j) \tag{14}$$

$$-\frac{\bar{u}_{j+1}^n - \bar{u}_{j-1}^n}{2h} + \bar{u}_j^n = 0,$$

and

$$\frac{\bar{u}_{N+1}^n - \bar{u}_{N-1}^n}{2h} + \bar{u}_N^n = 0. \tag{15}$$

Now, we subtract (9) from (13), we get

$$(\bar{U}^{n+1} - U^{n+1}) = A(\bar{U}^n - U^n) + \tau^n \tag{16}$$

We set $E^n = \bar{U}^{n+1} - U^{n+1}$, then the equation (16) becomes

$$E^{n+1} = AE^n + \tau^n \tag{17}$$

Clearly, E^n satisfies (4), we have

$$E_k^{n+1} = b_kE_{k+1}^n + (1-2b_k)E_k^n + b_kE_{k-1}^n$$
$$+b_k\sum_{j=0}^{k-1} g_j[E_{k-j+1}^n - 2E_{k-j}^n + E_{k-j-1}^n] + \tau q_k^n + \tau^n \tag{18}$$
$$E_j^0 = 0, E_0^n = 0, E_N^n = 0$$

Since $\Lambda \le 1$, from equation (18), we get

$$\left|E^{n+1}\right| \le b_k\left|E_{k+1}^n\right| + (1-2b_k)\left|E_k^n\right| + b_k\left|E_{k-1}^n\right|$$
$$+b_k\sum_{j=1}^{k-1} g_j|[E_{k-j+1}^n - 2E_{k-j}^n + E_{k-j-1}^n\ ]| + \tau\left|q_k^n\right| + \tau^n$$
$$\le (b_k + (1-2b_k) + b_k + c\tau)\|E^n\| + \max_{1\le M\le n}\|\tau^M\| \tag{19}$$

where

$$\|E^n\| = \max_{1\le k\le N}|E_k^n|$$

and $|q_k^n| \le c,\ 0\le k\le N$, (c is constant) and $\sum\limits_{j=1}^{k-1} g_j \le 0$.

$$\left|E^{n+1}\right| \le (1+c\tau)\|E^n\| + \max_{1\le M\le n}\|\tau^M\|$$

$$\left|E^{n+1}\right| \le C\|E^n\| + \max_{1\le M\le n}\|\tau^M\|,\ (C = 1+c\tau)$$

$\|E^0\| = 0$, implies $\|E^n\| = 0$

$$\left|E^{n+1}\right| \le \max_{1\le M\le n}\|\tau^M\|$$

Since, $\lim\limits_{(h,\tau)\to(0,0)}\|\tau^M\| = 0, (1\le M\le n)$ implies that

$$\left|E^{n+1}\right| \to 0\ uniformly\ in\ \Omega\ as\ (h,\tau)\to(0,0).$$

$$\left|E_j^n\right| \to 0\ uniformly\ in\ \Omega\ as\ (h,\tau)\to(0,0).$$

The proof is completed.

V. NUMERICAL SOLUTIONS

In this section we obtain the numerical solution of the space fractional diffusion equation with mixed initial boundary conditions. To obtain the numerical solution of the space fractional diffusion equation (SFDE) by the finite difference scheme, it is important to use some analytical model. In this section we present an example in a bounded domain to demonstrate that SFDE can be applied to simulate behavior of a fractional diffusion equation by using Mathematica Software. We consider the following space fractional diffusion equation with mixed initial boundary conditions:

$$\frac{\partial u(x,t)}{\partial t} = d(x)\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} + q(x,t),\ 0<x<1,\ 1<\alpha<2,\ t>0$$

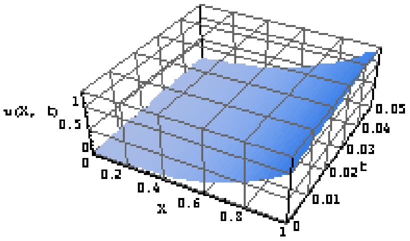
$$initial\ condition: u(x,0) = x^3, 0\le x\le 1$$

$$boundary\ conditions: -\frac{\partial u(0,t)}{\partial x} + u(0,t) = 0,$$

$$\frac{\partial u(1,t)}{\partial x} + u(1,t) = 0,\ t>0$$

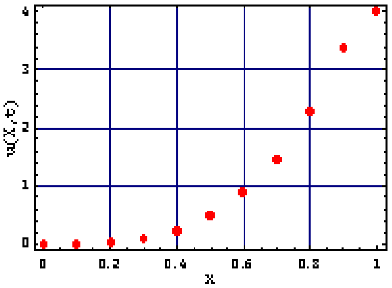
with the diffusion coefficient $d(x) = \frac{\Gamma(2.2)x^{2.8}}{6}$ and the source or sink function $q(x,t) = (1+x)e^{-tx^3}$. The numerical solution obtained by considering the parameters $\tau = 0.005$, $h = 0.1$, $\alpha = 1.8$, $d(x) = 1$ and $d(x) = \frac{\Gamma(2.2)x^{2.8}}{6}$ which is simulated in the following figures.

Surf-1.png



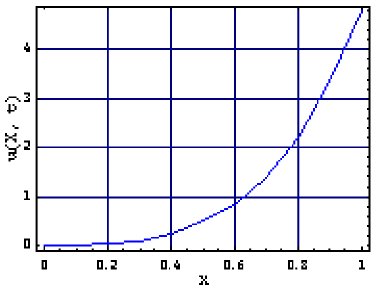
$Fig.5.1$: Analytical solution of diffusion equation with $t = 0.05,\ h = 0.1,\ \alpha = 2\ and\ d(x) = 1.$

RBC-22.png



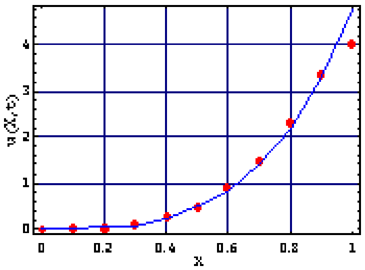
$Fig.5.2$: Temperature profiles along x direction at $\alpha = 1.8$, $\tau = 0.005,\ h = 0.1,\ and\ d(x) = 1$

RBC-33.png



$Fig.5.3$: Temperature profiles along x direction at $\alpha = 1.8$, $\tau = 0.005,\ h = 0.1,\ and\ d(x) = \frac{\Gamma(2.2)\times x^{2.8}}{6}$.

RBC-44.png



$Fig.5.4$: Comparison of temperature profiles along x direction at $\alpha = 1.8,\ \tau = 0.005,\ h = 0.1,$ $d(x) = 1\ and\ d(x) = \frac{\Gamma(2.2)\times x^{2.8}}{6}$.

May 07, 2012

VI. CONCLUSION

(i) We develop the new space fractional order explicit finite difference scheme for SFDE in a bounded domain.

(ii) The numerical example is presented to show that the numerical results are in good agreement with our theoretical analysis.

(iii) Therefore, our explicit finite difference scheme is numerically stable.

(iv) The explicit finite difference scheme required much less CPU time than an implicit schemes because the solution of system of algebraic equations does not involves the inversion of higher order matrices.

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